Removing Lines in Teaching Deductive Geometry

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Adding extra lines is unquestionably a useful skill when solving geometry problems. However, removing lines is also an important tactic. Key characteristics can be more easily seen after removing lines. In this article, I would like to share my experience using the technique described above to teach a lesson at the Form 3 level.

Medians and Centroid

For any given triangle, the three medians are concurrent, and the centroid trisects the medians. i.e. In Figure 1, AL, BN and CM are medians of ΔABC. These three lines intersect at G, the centroid of ΔABC. Then we have, $\frac{LG}{AG} = \frac{NG}{BG} = \frac{MG}{CG} = \frac{1}{2}$. It is not difficult to present the proof to Form 3 students. However, how to lead the students to discover the proof?

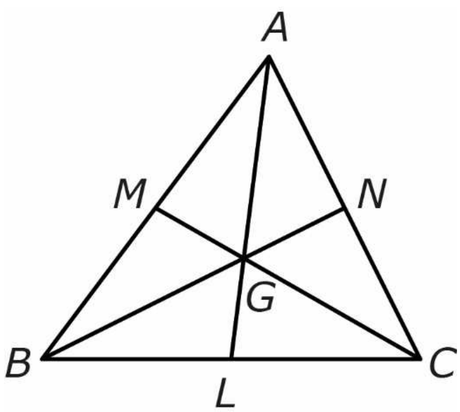

Figure 1

Below is a lesson snapshot. The lesson was conducted after the students investigated the properties of centroid by paper-fold in the previous lesson. After that activity, students conjectured about the concurrence and trisection.
(Teacher presented Figure 1 on the whiteboard.)

Teacher: "What have you discovered in the figure?"

(Students shared their findings. They still had no idea how to prove the conjecture. Meanwhile, I gave them a hint.)

Teacher: "How about we remove line *AL* from the original graph and consider a simpler figure? (Clear the line *AL* on the whiteboard.) What is the diagram making you think of?"

Student A: "Mid-Point Theorem!"

(Students quickly realized that they could use the Mid-Point Theorem, where they learned it in the previous chapter since the removal of line *AL* reduces the number of mid-points from three to two. But I followed up with questions to make sure the students knew how to use the theorem.)

Teacher: "Very Good! Why do you think the Mid-Point Theorem might be used?"

Student A: "There are two mid-points!"

Teacher: "Could you be more specific?"

Student A: "*M* and *N* are the mid-points of *AB* and *AC*. We can join *MN*. (Figure 2) Now, we can apply the Mid-Point Theorem. The results are $2MN = BC$ and $MN \parallel BC$."

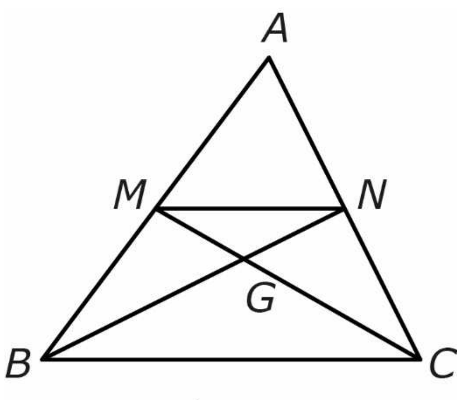

Figure 2

Teacher: "Excellent! We make a major step forward! Next, what? How do we proceed?"

Student B: "Actually, the result $2MN = BC$ suggests $MN : BC = 1 : 2$. It is very similar to what we are going to prove…"

Student C: "$\Delta MGN \sim \Delta CGB$! $\frac{MN}{BC} = \frac{1}{2}$, therefore $\frac{NG}{BG} = \frac{1}{2}$ and $\frac{MG}{CG} = \frac{1}{2}$."

Teacher: "Nice! Could you please verify your claim that $\Delta MGN \sim \Delta CGB$?"

It was spoken aloud by Student C and a few others.

"In ΔMGN and ΔCGB,

$\angle MGN = \angle CGB$ (vert. opp. ∠s)

$\angle GMN = \angle GCB$ (alt. ∠s, $MN // BC$)

$\angle GNM = \angle GBC$ (alt. ∠s, $MN // BC$)

$\Delta MGN \sim \Delta CGB$ (A.A.A.)

$\frac{NG}{BG} = \frac{MN}{BC} = \frac{1}{2}$ and $\frac{MG}{CG} = \frac{MN}{BC} = \frac{1}{2}$ (corr. sides, ~Δs)."

Teacher: "Good work! How about the median *AL*, though?"

Student A: "I think we can repeat the same trick again! Similar reasoning can be used to prove that $\frac{LG}{AG} = \frac{1}{2}$ by removing the line *CM* (in Figure 1). At last, there is $\frac{LG}{AG} = \frac{NG}{BG} = \frac{MG}{CG} = \frac{1}{2}$."

Teacher: "Brilliant! But how can you explain that the **point *G*** you just named and the point *G* in Figure 2 are identical? In other words, does the lines *AL*, *BN* and *CM* intersect at the same point?"

(Student A implicitly assumed that the medians are concurrent in his proof! This was what we were going to prove. I gave them further instructions to make things clear.)

Teacher: "To avoid confusion, let us call the point of intersection of *AL* and *BN* be *G'*. Using a similar argument, Student A has just shown that $\frac{LG'}{AG'} = \frac{1}{2}$ and $\frac{NG'}{BG'} = \frac{1}{2}$. How can we show that *G* = *G'* ? Can we carefully examine these findings made by Student A and Figure 2 to find any clues?"

(After some discussions…)

Student D: "The results $\frac{NG}{BG} = \frac{1}{2}$ and $\frac{NG'}{BG'} = \frac{1}{2}$ suggest *G* = *G'*! 1 Therefore

$$\frac{LG}{AG} = \frac{NG}{BG} = \frac{MG}{CG} = \frac{1}{2}$$."

Teacher: "Good! From the above proof, we can see that the medians intersect at the same point. We say that the three medians are **concurrent**. We also show that each median is divided in the ratio 1: 2 by this intersection point, which we will refer it as the **centroid**. Well done, students!"

Conclusion

The approach of removing lines enables students to recognize the diagram's key components. Because the figure is comparable to what they have already seen, it aids students in recalling earlier deductive geometry knowledge. After the line *AL* is removed and *MN* is added, the Mid-Point Theorem and similar triangles are immediate consequences. There is no necessity that teachers fill out every detail of the proof. Most of the proof can be found by the students themselves.

Remark

The following explanation might be used to further explain the result implies $G = G'$ for the students:

$$\frac{NG}{BG} = \frac{NG'}{BG'}$$

$$\frac{NG}{BG} + 1 = \frac{NG'}{BG'} + 1$$

$$\frac{NG+BG}{BG} = \frac{NG'+BG'}{BG'}$$

$$\frac{BN}{BG} = \frac{BN}{BG'}$$

$$BG = BG'$$

Since points *G* and *G'* are both located between points *B* and *N*, *G* = *G'*.

References

French, D. (2004). *Teaching and learning geometry* (pp. 83 – 84).

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