Mathematics Textbook analysis - a Snapshot of Textbooks in Two Different Curricula

Eugene Sze

中文摘要：本文以比較香港現有兩個課程——香港中學文憑試（DSE）及國際文憑試（IB）課程所擬定的教科書為研究對象，比對兩個課程教科書當中對微積分課程的異同。本論文比較兩個課程的教科書中教材內容的結構、教材中例子的運用及所提供的練習題目等。本文的結論提出兩個課程的教科書皆能有效的對應及實踐到該課程教學大綱所提出的學習目標。其次，本研究認定香港中學文憑試或國際文憑試的教科書為代表東方及西方的教育精神，發現當中國際文憑試的課程相比香港中學文憑試，具備更多「真實的」應用問題，這個結論與 Fan & Zhu（2006）的研究一致。

Abstract

This study compares two selected textbooks; one designed for the Hong Kong Diploma of Secondary Education (DSE) curriculum, the other for the International Baccalaureate (IB) Diploma curriculum. In terms of content analysis, the treatment of differential calculus has been selected for the study to illustrate the differences and similarities of the curricula. Emphasis has also been placed on analysing the similarities and differences in terms of the content structure, examples provided, and the type of exercises available. The study concludes that both textbooks serve well as the potentially implemented curriculum for both curricula, since they support all of the respective learning objectives found in the official syllabi. In this study, the IB and DSE textbooks were treated as representatives of Western and Eastern educational ethos respectively. In this regard, the former was found to include more “authentic” application problems than the latter, as consistent with the study by Fan and Zhu (2006).

Introduction

In mathematics education, textbooks are an integral part of teaching and learning. Textbooks can serve as a bridge between the intended curriculum and the implemented curriculum (Johansson, 2005). The former refers to an education system's aim and goals, immersed in the local culture, often written by the official government institute responsible for education in a particular region. The latter refers to the curriculum at the teacher's level: how activities are planned and employed in lessons. In a curriculum model by Valverde et al. (2002), textbooks, along with other organised resources, serve as the 'potentially implemented curriculum' that links together the intended and implemented curriculum. The extent to which teachers place their trust in textbooks is such that frontline teachers can, and often do, rely on textbooks as the curriculum without further searching for the curriculum guide from official documents of the state. In Hong Kong's case, that would be the Curriculum and Assessment Guide from the Education Bureau (CDC-HKEAA, 2015).

The textbooks being analysed in this study are for two different courses. One aimed for the local Diploma of Secondary Education (DSE) curriculum; the other is designed for the International Baccalaureate Diploma programme (IBDP). Not many studies are done that compares these two curricula, particularly in the context of their respective mathematics textbooks. With the DSE, the text is designed for Module 2 (Algebra and Calculus), written by Tang, Tam, Chan, Mui, Lo and Lo (2014). With 61 hours of suggested teaching hours devoted to the Calculus area of this Module (CDC-HKEAA, 2015), a textbook designed for the Higher Level curriculum of the IBDP was chosen for comparison, with a suggested 48 hours of instruction (IB, 2012) from the IB. The IB textbook is written by Martin, Haese, Haese, Haese and Humphries (2012). In particular, the area for differential calculus is an interesting area of study in terms of comparison of textbooks. The two syllabi are almost identical in terms of mathematical content; the main difference lying in the need for evaluating derivative of inverse trigonometric functions, as well as using calculus for kinematics problems for the IB. The target audience for the DSE Module 2 are those who "are more able in mathematics, or need more mathematical knowledge and skills to prepare for their future studies and careers" (CDC-HKEAA, 2015, p. 14), whereas those for the IBDP HL targets students who “will be expecting to include mathematics as a major component of their university studies” (IB, 2012, p. 5). The two courses therefore are designed with similar types of students in mind.

Another major difference between the DSE and IBDP is evident in their intended curriculum as outlined in the Curriculum and Assessment Guide for the DSE and the Mathematics HL Guide for the IB. The IB makes it explicit where connections with other areas of the curriculum can be made; these are listed alongside the content to be taught. Whereas for the DSE, there is a section titled ‘Cross-curricular Links’ (CDC-HKEAA, 2015, p. 92) in their guide that have a few suggested linkage with other subjects and associated activities. These suggestions are neither specific nor incorporated well into the syllabus. There are two main incentives for the IB teaching and learning to include cross-curricular links. One is the compulsory ‘internal assessment’ component of the course where students have to complete an exploration of mathematics that could be based on the applications of mathematics in other areas of their study. The other is due to the design of the IB Diploma. In order to attain the Diploma, students have to complete ‘core’ components as well as their ordinary subject-based curriculum. One aspect of this core is known as Theory of Knowledge (ToK). This is a timetabled subject and teaches ways of thinking and ways of knowing; rather close to what one will learn in philosophy. It is required by ToK that students have to link these aspects of how knowledge is attained within academic subject disciplines. As such, the curriculum guide provides ample samples of how these can be addressed in teaching mathematics. For instance, in the introduction to differential calculus, the ToK linkage specified in the curriculum guide includes the following: “Does the fact that Leibniz and Newton came across the calculus at similar times support the argument that mathematics exists prior to its discovery?” (IB, 2012, p. 33). The IB textbook addresses this by including ample historical notes related to the topics at hand. Moreover, ToK sections are included where appropriate. For instance, in the section that introduced limits, the ToK section refers to paradoxes that relate to limits (Figure 1).

THEORY OF KNOWLEDGE

The Greek philosopher Zeno of Elea lived in what is now southern Italy, in the 5th century BC. He is most famous for his paradoxes, which were recorded in Aristotle's work *Physics*.

The arrow paradox

"*If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless.*"

This argument says that if we fix an instant in time, an arrow appears motionless. Consequently, how is it that the arrow actually moves?

The dichotomy paradox

"*That which is in locomotion must arrive at the half-way stage before it arrives at the goal.*"

If an object is to move a fixed distance then it must travel half that distance. Before it can travel a half the distance, it must travel a half *that* distance. With this process continuing indefinitely, motion is impossible.

Figure 1: A ToK section in the text (Source: Martin et al., 2012, p. 517)

In this regard, the textbook being analysed is very much in line with the intended curriculum of the IB. In the DSE text, these curriculum links are entirely absent, thus leaving the cross-curricula links up to individual teachers to integrate into their teaching.

The structure of the textbooks

The first impression for both of these textbooks is the physical outlooks. The IB textbook carries all of the content in one volume, producing a single volume that is close to 1000 pages. On the other hand, the DSE textbook prints its units in booklet form. This allows students to carry small units at a time. In terms of practice exercises, the IB textbook has all the exercises included in the text. The DSE has supplementary exercises as a separate book that is available for purchase. In terms of practicality, the IB text is much more of a burden for students to carry to and from their schools. To address this, they include online access in their textbook package that includes the main text, as well as material that supports the use of Graphics Display Calculators (GDC) and some other supplementary material such as a chapter on the prerequisites to the course, for the duration of the course. Both textbooks are colour printed; both uses colour boxes to highlight important notes.

Both textbooks have their own style in terms of structuring a particular chapter. For the DSE text, they appear to follow the common organisation of a textbook; the 3e's model: ‘exposition, examples, exercises’ (Love and Pimm, 1996). Each chapter begins with the learning objectives followed by sections named ‘Review and ‘Warm-Up Exercise’. These sections aim to establish whether students have acquired the basic techniques needed for the chapter in question. The former describes the basic techniques and the latter provides opportunities for self-assessment. These are effective starting points to a chapter and allow students to be prepared for what follows in the rest of the text. It also provides a starting point for weaker students to engage with the required skills and review appropriate content prior to tackling the content of the current chapter. The learning objectives found here at the start of the chapter are drawn from the intended curriculum. They are often expansions of what is listed in the curriculum guide. This implies that by following the learning objectives of the textbook, users will have covered all learning objectives listed in the curriculum guide. For example, learning unit 6 in the DSE curriculum guide includes the following as learning objectives: “understand the intuitive concept of the limit of a function” and “find the limit of a function”. In the textbook, these are included along with "Recognize the definitions and notations of the number *e* and the natural logarithm." in the Learning Objectives.

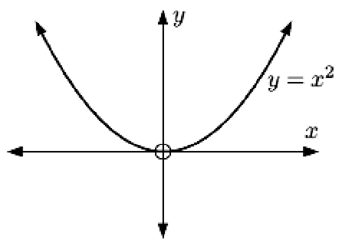
What follows are the examples with their solutions, along with explanatory material outlining the process. A small number of practice questions, usually one to three, can be found after each example for quick concept checks. Each section then ends with exercises aimed at consolidating what has been taught.

In contrast with the 3e's model, the IB textbook follows the structure of the ACE model: “Activities, Cours, Exercise” (Pepin and Haggarty, 2001). This model originates from French textbooks, as suggested by the French word “Cours”. Here, activities refer to guided activities, often investigative in nature, that introduce the concepts to the students. Cours refers to the content to be taught in explanatory words along with worked examples. The IB textbook does not provide the same introduction to a chapter as the DSE text, although they do include the names of the topics in a chapter and reference to the official syllabus guide. The IB places emphasis on inquiry-based learning, not only at the Diploma level, but also within primary and lower secondary school programmes, called the Primary Years Programme (PYP) and the Middle Years Programme (MYP). All the programmes encourage discovery and student-led activities, with the internal assessment component being evidence of this ethos. Their textbooks, including the one used in this study, reflect this theme. In order to aid students in developing a more open mind and encouraging directed self-learning, the chapters begins with an Opening Problem (Figure 2) for students to examine before the topic begins.

OPENING PROBLEM

Consider the curve $y = x^2$.

In the previous chapter we found that the gradient function of this curve is $\frac{dy}{dx} = 2x$.



Things to think about:

a Consider the transformation of $y = x^2$ onto $y = x^2 + 3$.
- **i** What transformation has taken place?
- **ii** For a given value of x, has the gradient of the tangent to the function changed?
- **iii** What is the gradient function of $y = x^2 + 3$?

DEMO

b Consider the transformation of $y = x^2$ onto $y = (x-2)^2$.
- **i** What transformation has taken place?
- **ii** How does the gradient function of $y = (x-2)^2$ relate to the gradient function of $y = x^2$?
- **iii** Can you write down the gradient function of $y = (x-2)^2$?

Figure 2: Opening Problem of the IB textbook
(Source: Martin et al., 2012, p. 530)

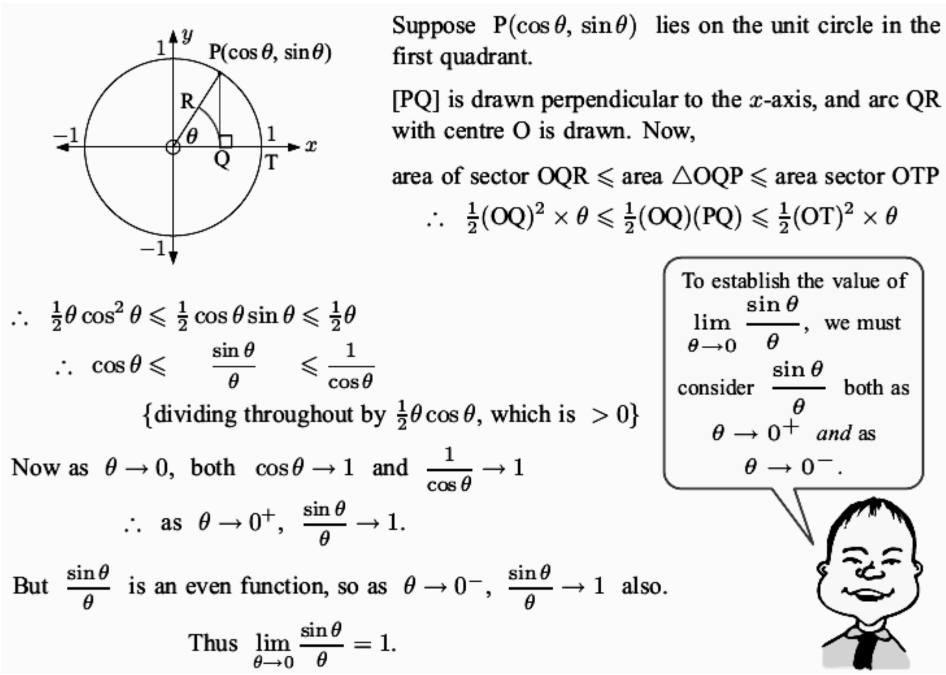
Subsequent exercise questions in the chapter often come back to this opening problem and ask for a solution after the students have been exposed to the content. These opening problems aim to foster a thinking model for students, in which they analyse the most effective way to solve unseen problems, while providing a thinking framework to help students structure their mathematics exploration in their internal assessment.

At the end of a chapter, the DSE text provides a chapter summary that includes facts that are deemed essential for students.. Such a summary is missing from the IB textbook. The inclusion of a summary is a major advantage that the former has over the latter, since it enables students to quickly review material when they prepare for assessments. Both textbooks offer review and revision exercises at the end of each chapter. They provide mixed-skills exercises that allow students to apply all knowledge covered in that particular chapter. IB examinations are broken down to 3 papers: Paper 1 and 3 are GDC dependent and paper 2 is a non-calculator paper. As such, these review exercises are organised this way in their textbook; there are review sets aimed at non-calculator practice as well as ones that aim at practicing GDC usage.

Treatment of introducing differential calculus

In preparation for the introduction of differential calculus, both curricula called for an informal understanding of limits. Understanding of the notion of limits is an important watershed in one's mathematics education. Some researchers argue that an informal understanding of limits is a necessity in order for one to understand the formal definition of limits (Cottrill et al., 1996). Even though rigorous proof of limits are not required at this stage, the DSE text offers a much more in depth exploration of the notion of limits compared to the IB text. It is worth to note here that the IB provides an optional unit called ‘Calculus’ where more rigorous study of limits is taken. The difference seems to commensurate with the intended curriculum. The IB asks for “informal ideas of limit, continuity and convergence” (IB, 2012, p. 33), whereas the DSE asks to find limits of a function on top of the “intuitive concept of the limit of a function” (CDC-HKEAA, 2015, p. 58). From experience, I know that the implemented curriculum for the IB varies from the intended curriculum here. Since the intended curriculum only asks for an informal definition and understanding, it is difficult to assess in assessments. Moreover, teachers who teach to tests do not require students to understand limits fully in order to teach the mechanics behind differentiation. By explicitly placing finding limits as part of the intended curriculum, the DSE has made it examinable and the textbook has devoted a chapter to deal with this topic; the IB text only has 3 subsections within a chapter for limits.

Both textbooks listed some rules and properties of finding limits, however neither attempted to prove these properties, since they involve strict definition of limits. Even though the scope of both intended curricula excludes looking at limits approaching from the left and right, both texts include a section explaining the concept. This provides some mathematical rigor for students, enabling them to build on their existing understanding. This would be particularly pertinent if they were to take a mathematics course in university, as these early concepts could help overcome possible misconceptions. This concept is useful for showing proofs for $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$, which is present in both intended curricula. Both proofs implicitly uses the squeeze theorem, but the IB text is more elegant and dispenses with the need to provide further proof that $\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$is valid (Figure 3).


Suppose $P(\cos\theta, \sin\theta)$ lies on the unit circle in the first quadrant.

[PQ] is drawn perpendicular to the x-axis, and arc QR with centre O is drawn. Now,

area of sector OQR $\leqslant$ area $\triangle$OQP $\leqslant$ area sector OTP

$\therefore \ \frac{1}{2}(OQ)^2 \times \theta \leqslant \frac{1}{2}(OQ)(PQ) \leqslant \frac{1}{2}(OT)^2 \times \theta$

$\therefore \ \frac{1}{2}\theta\cos^2\theta \leqslant \frac{1}{2}\cos\theta\sin\theta \leqslant \frac{1}{2}\theta$

$\therefore \ \cos\theta \leqslant \frac{\sin\theta}{\theta} \leqslant \frac{1}{\cos\theta}$

{dividing throughout by $\frac{1}{2}\theta\cos\theta$, which is > 0}

Now as $\theta \to 0$, both $\cos\theta \to 1$ and $\frac{1}{\cos\theta} \to 1$

$\therefore$ as $\theta \to 0^+$, $\frac{\sin\theta}{\theta} \to 1$.

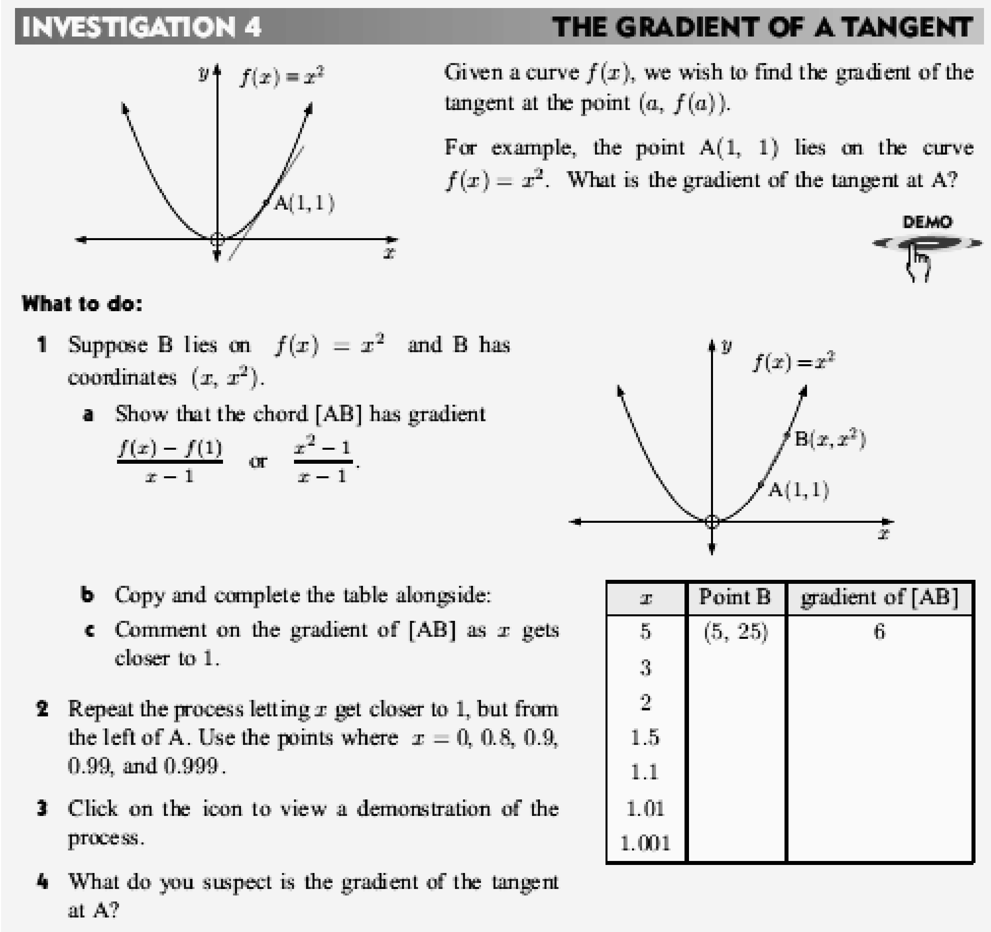
But $\frac{\sin\theta}{\theta}$ is an even function, so as $\theta \to 0^-$, $\frac{\sin\theta}{\theta} \to 1$ also.

Thus $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.

Figure 3: The proof from the IB text (Source: Martin et al., 2012, p. 516)

To introduce the notion of differentiation, both texts include the formal definition of differentiation by first principles, as listed in both curricula. However, the approach is different in both. The focus of the IB text is more on an application-based approach, whereas the DSE text goes directly to an abstract algebra-based approach. The IB text introduces the idea of "instantaneous rate of change" and encourages student-led investigative teaching by providing an investigation in context. This is then followed by another investigation that looks at gradient of a tangent by examining different chords forming from points on a curve. This investigation encourages students to look at the effect of the gradient of the chords as the points chosen gets closer and closer towards each other, mimicking the $\lim_{h \to 0}$ in the definition of the derivative. Students are encouraged to use their GDC while undertaking these investigations. In contrast, the DSE text has a small class activity that asks students to compute different slopes of chords with varying coordinates. While this does provide patterns for students to spot and emphasises the effect of small increments on the slope of a chord, it consists of simple calculations throughout. The IB exercises provide greater opportunities for critical thinking and allows students to have a real-life anchor to this abstract idea of derivative.

Concept image and concept definitions are terms proposed by Tall and Vinner (1981) in relation to the understanding of mathematics. Their focus was on undergraduates and their understanding of limits and continuity. From that study, Tall and Vinner described concept image as "all the cognitive structure associated with the processes, including mental pictures and associated properties and processes" (p. 152). In contrast, the concept definition is explained by the words used to explain that concept. These could be personal and do not have to coincide with the formal definition of that concept. Concept images are dynamic and can change with the students' experience in mathematics. The IB text has used a total of three investigative exercises prior to showing the processes of differentiation from first principles, an example of which is shown below:



INVESTIGATION 4 **THE GRADIENT OF A TANGENT**

Given a curve $f(x)$, we wish to find the gradient of the tangent at the point $(a, f(a))$.

For example, the point A(1, 1) lies on the curve $f(x) = x^2$. What is the gradient of the tangent at A?

What to do:

1 Suppose B lies on $f(x) = x^2$ and B has coordinates (x, x^2).
  a Show that the chord [AB] has gradient $\frac{f(x) - f(1)}{x - 1}$ or $\frac{x^2 - 1}{x - 1}$.
  b Copy and complete the table alongside:
  c Comment on the gradient of [AB] as x gets closer to 1.

x	Point B	gradient of [AB]
5	(5, 25)	6
3		
2		
1.5		
1.1		
1.01		
1.001		

2 Repeat the process letting x get closer to 1, but from the left of A. Use the points where $x = 0$, 0.8, 0.9, 0.99, and 0.999.

3 Click on the icon to view a demonstration of the process.

4 What do you suspect is the gradient of the tangent at A?

Figure 4: Investigation from the IB textbook
(Source: Martin et al., 2012, p. 520)

The outcome of each investigative exercise builds on the concept image of the students to develop a strong cognitive structure of the derivative process. This process begins with a contextualised image, moves towards the study of a single function and further develops the idea of gradient functions. The DSE text quickly delves into providing descriptive ‘steps’ into finding the slope of the tangent at a general point on a curve, followed by the formal definition. While this could strengthen the concept definition of a student, their concept image of the derivative process may not necessarily be strong. As suggested by Tall and Vinner, if there exists a conflict between one’s concept image and concept definition, misconceptions may occur. For instance, students of the DSE text may develop a strong concept definition of the derivative but their concept image of derivative may be of a strictly algebraic nature. This may make it harder for them to relate differentiation with their applications when they encounter them in later chapters or other subject areas. In terms of the mathematics involved, the DSE text's treatment is more complete than the IB for differentiation by first principles. As well as guiding students through the algebraic process of taking the limits to find derivative, they have introduced the idea of differentiability.

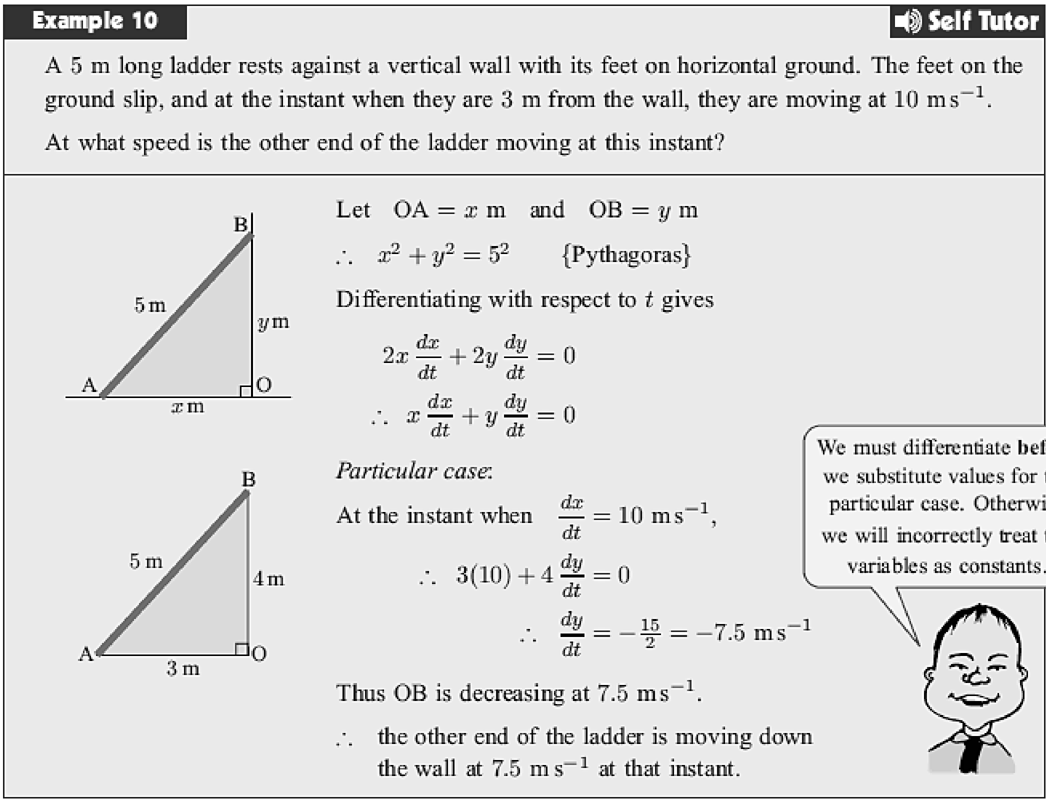
Presenting rules of differentiation

Similar differences in their approach exist in presenting differentiation rules. For the DSE text, they have opted for listing the basic rules on top of the product and quotient rules straightaway, followed by their proofs. While this is an effective treatment of the topic at hand, again it does not help students in developing their concept image. The proofs can appear to be an afterthought and only placed for the sake of completeness rather than a tool for learning. In contrast, the IB text allows room for students to investigate the patterns behind different derivatives. Instead of giving a formal proof in the beginning, they provide an investigative exercise that concludes in finding the rule for differentiating addition or subtraction of functions. The same is true for the product rule. In terms of academic rigor in mathematics, the DSE provide stricter proofs than the IB textbook. The IB textbook attempts at making the proofs easily understandable while the DSE text proves the rules by first principles. Each approach has its own merits and drawbacks. The DSE textbook's strict approach allows students a glimpse of how rigorous mathematics can be in the context of their knowledge content. The IB textbook's approach allows further development of students' concept image.

The theory of guided discovery in the investigative exercises is well intended and often well designed in the IB textbook. However, while the intentions are good, the implementation is difficult and this is partly due to the way the book is structured. In one investigation, students are supposed to use their GDC to manipulate the graph of $f(x) = b^x, b \in \mathbb{R}$ with its derivative graph to find when both graphs are identical. The aim is to figure that the derivative of e^x is itself. However, similar to other investigative activities in the text, the solutions can be quickly found by going to the next page (or in this case, simply by looking down the page). While completing the activity may enhance the student's concept image, the temptation to simply look at the solutions is too tempting and negates the feeling of a need to spend time and effort solving the problem. From experience, students are often impatient and tend to go directly to the results. This is one of the limits of textbooks in general: the attitudes of the students can influence how effective a textbook is to the learners. It can also be argued that these are not discovery exercises since the destination is set and the authors are actively directing the students to their pre-set destination.

Examples in the textbooks

Examples are an important part of a textbook. They allow students to mimic how mathematical problems are solved and provide a framework to model on when students try to tackle problems themselves. There are 4 basic stages of problem solving that Pólya (1957) has stated: understanding the problem, formulating a plan, executing the plan and finally reviewing the processes. Useful examples should include all these stages to demonstrate the thinking process as well as the resulting solution. In the applications of differentiation chapters of both textbooks, there are clearly stated stages, including a step-by-step guide on how to solve problems. However, this appears to be the second stage of problem solving and does not directly aid students in understanding the problem at hand. A useful aid at this stage is a monologue-style thinking instruction which demonstrates how the problem should be understood. In terms of showing the plan execution stage, the IB textbook does a better job at explanation. There are often speech bubbles to highlight possible points for errors as well as written explanations of what is being done. These hints are not found in the DSE text. Another additional advantage of the IB text is the way they structure their worked response: a series of diagrammatic representations are present within some solutions, if they are deemed to help clarify the question at hand.



Example 10 — **Self Tutor**

A 5 m long ladder rests against a vertical wall with its feet on horizontal ground. The feet on the ground slip, and at the instant when they are 3 m from the wall, they are moving at 10 m s^{-1}.

At what speed is the other end of the ladder moving at this instant?

Let OA $= x$ m and OB $= y$ m

$\therefore\ x^2 + y^2 = 5^2$ {Pythagoras}

Differentiating with respect to t gives

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\therefore\ x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Particular case:

At the instant when $\frac{dx}{dt} = 10$ m s^{-1},

$$\therefore\ 3(10) + 4\frac{dy}{dt} = 0$$

$$\therefore\ \frac{dy}{dt} = -\tfrac{15}{2} = -7.5 \text{ m s}^{-1}$$

Thus OB is decreasing at 7.5 m s^{-1}.

$\therefore$ the other end of the ladder is moving down the wall at 7.5 m s^{-1} at that instant.

Figure 5: An example with a cartoon teacher's hint
(Source: Martin et al., 2012, p. 618)

These sequential diagrams, such as those in Figure 5, can aid students in visualising what is occurring at different stages of their problem solving. To include Pólya's final stage of problem solving, both textbooks could include a paragraph and working on how to check whether the answers achieved is reasonable in the context of the question. If the question has no context and is purely mathematical, hints should be given as to how the solution can be used within the question to check whether it is reasonable. Another element that is missing from the worked examples is an alternative method to solve these problems. Wong el al. (2002) found that some students have a rather narrow view of mathematical problem solving. They believe that there is one “correct” way to solve a problem and that there is always a unique solution. While the nature of assessments would almost guarantee that the latter has to be true, there exists multiple approaches to problem solving, even for examination questions. Textbooks therefore should include alternative methods to solving problems, particularly for problems that do not involve routine algorithmic procedures.

Exercises in the textbooks

Exercises can arguably define the strength of a textbook. If questions are too simple, students will be unprepared for assessments. If the questions are too difficult, they are unnecessary and can discourage otherwise competent students. Too little in terms of question numbers would mean students lack the resources needed for consolidating content learnt. The exercises on both textbooks vary in terms of structure. In the DSE text, the exercises are graded in terms of difficulty. The simpler ones are named Level 1 and the more complex ones Level 2. Level 1 problems are usually closely related to the worked examples prior to the exercise. In contrast, the IB text's exercises are simply a collection of problems without grades. However, they usually get progressively more difficult. The style of the questions is influenced by the assessment methods. As previously mentioned, GDC is allowed in the IB examinations. Some questions there guide students to effectively use their GDCs (Figure 6)

3 Find the gradient of the tangent to:

a $y = x^4(1-2x)^2$ at $x = -1$

b $y = \sqrt{x}(x^2 - x + 1)^2$ at $x = 4$

c $y = x\sqrt{1-2x}$ at $x = -4$

d $y = x^3\sqrt{5 - x^2}$ at $x = 1$

Check your answers using technology.

4 Consider $y = \sqrt{x}(3-x)^2$.

a Show that $\frac{dy}{dx} = \frac{(3-x)(3-5x)}{2\sqrt{x}}$.

b Find the x-coordinates of all points on $y = \sqrt{x}(3-x)^2$ where the tangent is horizontal.

c For what values of x is $\frac{dy}{dx}$ undefined?

d Are there any values of x for which y is defined but $\frac{dy}{dx}$ is not?

e What is the graphical significance of your answer in **d**?

Figure 6: Exercise leading students to use their GDC

(Source: Martin et al., 2012, p. 541)

Other than specifically being in an ‘application’ chapter, the DSE textbooks tend to offer questions that are calculations based with little variation in the way the questions are asked. Their focus seems to be on getting students to be proficient at performing the algorithms and rules for differentiation. It does provide plenty of opportunity for students to practice questions of a certain type.

While the DSE's questions are calculations based, the IB's provide more room for thought beyond calculating derivatives. They also expose students to more ways in which questions can be asked.

Styles aside, both textbooks are giving students similar number of practice questions. The summary below (Table 1) is the number of worked examples and exercises in the “rules of differentiation” portion of the textbooks, excluding the review sections. These numbers do not take into account supplementary exercises available for the DSE text. It should also be noted that the IB textbooks include a greater number of multi-part questions that are counted as single questions. The result suggests that students of both textbooks can experience a similar amount of practice. The proportion of worked examples to exercise questions seems to be similar in both textbooks as well. This means that students would have similar exposure to ways of solving a particular type of question prior to tackling the exercise. Because of the way the questions are asked in the IB textbook, it is not possible to cover all types of questions being asked in the exercises: more understanding and thinking is required than the DSE counterpart, where rote learning could allow the completion of most of the exercises. Both textbooks have the expectation that their proportion of worked examples would allow students to generalise to other cases. In the Teacher’s Edition, the DSE textbook provides additional examples should the ones found in the textbook be found to be insufficient for the students.

Table 1: A table showing number of worked examples against number of exercise problems in the ‘rules of differentiation’ chapter

	DSE Textbook		IB Textbook	
	Worked examples	Exercises	Worked examples	Exercises
Number of problems	35	289	34	255
Percentage	10.8%	89.2%	11.8%	88.2%

Fan and Zhu (2006) have found that when comparing textbooks from China against those from the United States, the Western textbook have more applications-based questions when compared to an Eastern textbook. However, the definition of “application” is debatable. In the area of our interest, there is specifically a chapter on application of differentiation in both textbooks. In particular, the sections on optimisation problems and related rates of change problems are looked at. This is because these areas lend themselves naturally to application problems, where solutions as a result of solving these questions can have a direct impact in real-life. The other type of “application” question that is also present in most textbooks are those that *uses* the concepts being taught, but not applying mathematics in a real-life context. Many questions are simply making a narrative to contextualise the mathematical question. The danger here is that students are being led to essentially ignore the context and focus on the numbers and symbols. The methodology for the following tally is quite subjective and is used to highlight the difference in content. The term “authentic” application questions are used to refer to question with solutions that can be useful in real-life; “non-authentic” refers to those that have a superfluous narrative and are simply there as a contextualisation tool. The counts were done by the author and a peer without further reliability checks. The results however tell a story that supports the findings of Fan and Zhu. The Western IB text, published in Australia, contains significantly more authentic problems than the Eastern DSE text, published in Hong Kong. While some of these authentic questions are still fictitious, they are presented in a way that suggests how the mathematics being learnt at this stage could be applied in a real-life situation. It would benefit students if there could be real-life case studies of how differentiation can be applied with real-life data to expand students' mathematical horizon.

Table 2: Table showing number of “authentic” and “non-authentic” application problems in the applications of differentiation chapters

Applications of differentiation	DSE Textbook		IB Textbook	
	Non-authentic	Authentic	Non-authentic	Authentic
Number of problems	41	6	36	14
Percentage	87.2%	12.8%	72%	28%

The audience of the texts

The primary intended readers of the textbooks are students. However, they are not the sole readers, with teachers being the secondary intended readers of the texts. In order to use the textbooks efficiently, teachers need to be familiar with the content of the text and ensure there are no direct conflict between the textbook and what is being said in class. From experience, students can see the textbook as a type of “authority-figure”. If there are discrepancies between what is being taught and what is printed, the resulting “but the textbook says this” is almost inevitable. It could be a matter of alternative representations or equivalent forms. This is especially true when students undergo self-assessment by checking their answers to problems with the solutions provided at the back of both textbooks. The textbooks often give only one form of representation in their answers and when it differs from their own solution, they question their own correctness and interpretation of the text. This is where the role of a teacher cannot be simply replaced by a textbook. The teacher could quickly point out why the difference is present, or it could even be that there is an erratum in the text, where the textbook cannot. There can also be a mismatch between what the authors intend students to grasp and what students glean from the text. Textbooks attempt to deal with this mismatch by providing worked examples. In addition, the DSE textbook has a Teacher's Edition that includes teaching notes to guide teachers to follow the authors' intentions. The IB text does not have a teacher's edition. They have built their accessories around making the text as stand-alone as possible. Animated worked examples with voiced guidance and explanation are available to students on the attached CD. Moreover, as seen in excerpts of the IB textbook earlier in the study, they have intermittent speech bubbles from cartoon characters acting as 'teachers' with hints and explanations throughout the text.

Summary and discussions

Overall, both textbooks are effective tools as the potentially implemented curricula. Both textbooks match well to their respective learning objectives in the official curriculum guide. If they were used as intended by the authors, both could lead to the implemented curricula being identical to the intended curricula. The mathematics in both textbooks is rigorous enough for their purposes. Other than the obvious differences in their syllabus, the textbooks can be used interchangeably on each other’s curriculum.

In terms of preparing students for their respective examinations, both textbooks do a fairly good job. They are both capable of guiding students to form strong concept images of the topic at hand. There are plenty of exercises provided to consolidate learning. However, there is a type of questions that is less addressed on the DSE text for this particular topic. Both curricula include ‘explain your answer’ type questions in their final assessment. While the IB text includes plenty of practice questions of this type (take Figure 6, question 4e as an example), the calculations-based exercises provided by the DSE text prepares students less. The emphasis is again placed on teachers. In their use of the textbook, teachers should be aware of the objectives of the assessments and ensure that students are adequately prepared to express their thinking processes in order to answer these questions.

It is noteworthy that this study is a snapshot of the issue, having focused on one area of the curricula and only one publisher from each. It is possible that other publishers have better treatments of the topic at hand and that the exercises involved are better designed. It is also possible that other areas of the syllabi are less well matched in the textbooks against the respective intended curricula. To affirm or deny either proposition would require a closer look at different publishers and across a wider range of topics of the two curricula. The outcome of such a study could help inform curriculum development as well as developing better use of mathematics textbooks.

In terms of curriculum development, such a study can shed light on the pros and cons of textbooks and on their suitability to their respective curricula. Ideas and good practices could be taken from other syllabi to supplement existing structures. The fact that an increasing number of schools in Hong Kong have taken up the IB as either their sole pre-university curriculum or one that runs alongside the DSE (Yau, 2015), can be an indication that Hong Kong's education system is environmental friendly for the adoption of the IB curriculum. This suggests that outcomes from a study that comprehensively compares the textbooks from the two different curricula could allow local policy makers to identify what is appealing to mathematics educators in other contexts and include components to our existing syllabus where appropriate.

References

Cottrill, J., Dubinsky, E., Nichols, D., Schwingendorf, K., Thomas, K., & Vidakovic, D. (1996). Understanding the limit concept: Beginning with a coordinated process schema. *Journal of Mathematical Behavior*, *15*, 167-192.

Curriculum Development Council and The Hong Kong Examinations and Assessment Authority (CDC-HKEAA). (2015). *Mathematics curriculum and assessment guide (Secondary 4-6)*. Hong Kong: Education Bureau.

Fan, L., & Zhu, Y. (2006). Focus on the representation of problem types in intended curriculum: A comparison of selected mathematics textbooks from Mainland China and the United States. *International Journal of Science and Mathematics Education*, *4*(4), 609-626.

International Baccalaureate (IB). (2012). *Mathematics HL guide*. Cardiff: International Baccalaureate Organisation.

Johansson, M. (2005). *Mathematics textbooks – the link between the intended and the implemented curriculum?* Paper presented at the Mathematics Education into the 21st Century Project: Reform, Revolution and Paradigm Shifts in Mathematics Education. Universiti Teknologi Malaysia, Johor Bahru, Malaysia.

Love, E., & Pimm, D. (1996). 'This is so': a text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education*, *1*, 371-409. Dordrecht: Kluwer.

Martin, D. C., Haese, R. C., Haese, S. H., Haese, M., & Humphries, M. (2012). Mathematics for the international student. Mathematics HL (Core) third edition. South Australia: Haese Mathematics.

Pepin, B., & Haggarty, L. (2001). *Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures.* Zentralblatt fuer Didktik de Mathematik, *33*(5), 158-175.

Pólya, G. (1957). *How to solve it*. Garden City, New York, U.S.A.: Doubleday and Co., Inc.

Tall, D. O., & Vinner S. (1981). Concept image and concept definition in mathematics, with special reference to limits and continuity, *Educational Studies in Mathematics*, *12*, 151-169.

Tang, M. C., Tam, C. F., Chan, M. H., Mui, W. K., Lo, Y. K., & Lo, M. T. Y. (2014). *Effective Learning Mathematics. Mmodule 2: Algebra and calculus.* Hong Kong: Chung Tai Educational Press.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002).

According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks. Dordrecht: Kluwer Academic Publishers.

Yau, E. (2015). IB Or DSE? Pros and cons of Hong Kong secondary school curriculums explained. *South China Morning Post*, 14 April. Web. 13 June 2016.

Wong, N. Y., Marton, F., Wong, K. M., & Lam, C. C. (2002). The lived space of mathematics learning. *Journal of Mathematical Behavior, 21*, 25-47.

Author's e-mail: sze.eugene@link.cuhk.edu.hk