對談講場

Dear Editor,
Thank you very much for the premier issue of EduMath.

The traditional procedures of expressing a number in bases other than 10 (Wong, *Teaching and Learning Mathematics...*, EduMath, Issue 1, pp.19-22) can be explained by recalling the Euclidean (or division) algorithm. Division of a positive integer a by another positive integer b means *repeated subtractions of b from a, say q times, till the nonnegative remainder, say r, is smaller than b.* To emphasize the division, we usually write

$$b \div a = q, \quad \text{remainder} = r,$$

instead of the more precise relation

$$a = b \cdot q + r, \qquad 0 \leq r \leq b.$$

Just as $4567 = 10^3 \cdot 4 + 10^2 \cdot 5 + 10 \cdot 6 + 7$, an expression in base 6 like 12345_6 stands for the number

$$\begin{aligned} & 6^4 \cdot 1 + 6^3 \cdot 2 + 6^2 \cdot 3 + 6 \cdot 4 + 5 \\ = \quad & 6\,[6^3 \cdot 1 + 6^2 \cdot 2 + 6 \cdot 3 + 4] + 5 \\ = \quad & 6[6[6^2 \cdot 1 + 6 \cdot 2 + 3] + 4] + 5 \\ = \quad & 6[6[6[6 \cdot 1 + 2] + 3] + 4] + 5. \end{aligned}$$

This simple calculation leads to the following observations.

(1)The number represented by 12345_6 can be evaluated *recursively* as follows:

	1	2	3	4	5
6		6	48	306	1860
	1	8	51	310	1865

(2)The number 1865 (in denary notation) can be converted into base 6 by *repeated divisions by 6*, the digits being the *remainders* in the successive divisions. This explains the traditional procedures.

It is certainly possible to rewrite this in a more general setting. For example, (1) above is exactly the same as the process of *synthetic division*. However, a concrete example like this should help beginners see what is going on here, and understand how things work.

Sincerely,
Paul Yiu

Editor's note to Maggie P.H. Wong's article "Teaching and learning mathematics with understanding: the case of converting denary numbers into binary numbers" (*EduMath, 1*, 19-22.)

Ms Wong's article provided teachers with useful recommendations to teach and learn the topic with understanding. Games to enhance the techniques of converting between denary and binary numbers can also be found (see, for example: Wong, N.Y. (1993). Games and mathematics education (in Chinese). ***Mathmedia, 66,*** 52-68.).

The topic was introduced into our school mathematics curriculum with the "modern mathematics" reform. "Representation of integers by means of different bases, including 2" was stated in the 1969 HKCEE Mathematics Syllabus B. That statement was excluded from the 1980 syllabuses (there were a number of HKCEE Mathematics syllabuses then). Now we have " conversion of denary into binary numbers and vice versa; addition and subtraction of binary numbers" in Topic 1.4 of Form 1 in the CDC Mathematics Syllabus (1985). One can notice from this a shift from the representation of integers in various bases, with the binary numbers being just one example among them, to number representations confined to base 2 only. There has been much discussion in the circle on whether this topic is too difficult and should be deleted from the syllabus altogether. Yet, the topic is still retained in the recent Mathematics Target-Oriented Curriculum (Curriculum Development Council (1992). Learning Targets for Mathematics.). We still need to wait and see if there are further amendments concerning this topic in the "Learning Targets for Key Stage 3 " which is still under preparation.

The editor takes the view that, in reducing the learning of integers from those in different bases (taking binary numbers as just one example) to just the binary numbers, learning is focused on the acquisition of the various skills involved. The concept of place value has therefore been de-emphasised. Though such a "downplay" seemingly makes learning easier (as contents are reduced), the reality may be just the opposite because it is more difficult then for students to see the whole picture and the relevance of the topic in the entire syllabus.

Similar cases can be found in other topics too when we try to cut down contents without due regard to the integrity and coherence of the reduced curriculum. For instance, with simplification of surds, when being treated just for simplifying expressions like $\frac{1}{\sqrt{3}-\sqrt{2}}$, the ingenious application of the formula $a^2 - b^2 = (a - b)(a + b)$ (the notion of conjugates) cannot be appreciated. In this connection, one may find interest in reading M.K. Siu's paper "Core curriculum = core syllabus? Core curriculum = second class curriculum?" (Paper presented at the Conference "Hong Kong Mathematics Education: A Time for Change?", 13 May, 1995, The Chinese University of Hong Kong).

In fact, it is hard to differentiate purely algorithmic skills from conceptual understanding in most cases (see, for example: Wong, K.M. (1994). Can mathematical rules and procedures be taught without conceptual understanding? ***CUHK Journal of Primary Education,*** *5*(1), 34-41.). And a reasonable position to hold is that learning of skills grows with conceptual understanding and vice versa. To teach calculation skills devoid of meaningful mathematical understanding is only doing disservice to mathematics learning in the long run.

第一期頁 14-15小敏「動『手』算一算」一文舉出一個有趣的算法，相信讀者都知道背後的原理了。其實袁文龍先生在《數學家族(1)》(劉應泉編，香港教育圖書公司，1995)的「手指計算機」一文便詳細闡述了箇中底蘊。有興趣之讀者可參考之。

A Brief Note on "Directed Numbers"

J.A.G. McClelland (1994) has made the following comment in his article "Junior secondary mathematics" (***Hong Kong Science Teachers Journal, 19*(1)**, 29-34.):

> The [Hong Kong mathematics] syllabus correctly refers to signed numbers but there is a tendency for authors to refer to 'directed numbers' when the sign of a number is introduced. I have not found this expression outside Hong Kong, but here it appears in both textbooks and articles (e.g. Fung [*sic*] & Siu 1992). Even if it does exist elsewhere, it is misleading, unnecessary and poor English. The expression seems to suggest that there are numbers with direction and numbers without direction. ... (p.30)

We would like to refer our readers to the following works where the term "directed numbers" does appear:

Maye, A.B. (1938). ***The Essentials of School Algebra.*** (see p.84)
Ferrar, W.L. (1943). ***Higher Algebra.*** (see p.57)

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第一期更正

1. 頁1行-8：「，能夠最終發展成爲本地」（漏去）
2. 頁5行17：應爲「如是我睹、我聞、我思」。
3. 「整數乘分數學教學一得」作者應爲滬江小學教師。
4. 頁13：編輯室按：「…Bruner, Gagne,…」應爲「…Bruner, Gagné,…」
5. 頁32行：「was by *no* means easy …」（漏去）

編委會謹此致歉