

Primary Japanese Mathematics in the First Decade of the 21st Century – the Course of Study ^(*)

Shotaro Tanaka
Tokushima Bunri University, Japan

Ngai-Ying Wong
The Chinese University of Hong Kong, Hong Kong

BACKGROUND

The National Council of Teachers of Mathematics of the United States has recently published its “*Standard-2000*” (*Principles and Standards for School Mathematics: Discussion draft*), and the California State Board of Education published its *Mathematics Content Standards for California Public Schools: Kindergarten through Grade Twelve*. In the United Kingdom, the Department of Education and Sciences and the Welsh Office of U.K. also published its *National Curriculum for the 21st Century (mathematics)*. In recent years, the mathematics curricula in various Asian countries have been undergoing revision too. In China, after developing a mathematics curriculum particularly for Shanghai in 1997, it now proceeds to developing a mathematics curriculum for the 21st century. In Taiwan, the current mathematics curriculum was published in 1994 and it has been under further revision in recent years. A high-level ad hoc committee on holistic review of the mathematics curriculum was established in Hong Kong in 1997 and the committee has just released its final report in December 1999. Malaysia, Singapore and South Korea also revised their mathematics curricula in 1993, 1998 and 1999 respectively. In Japan, a new curriculum for primary mathematics is published this year. This is another focal point for mathematics educators and curriculum developers in Asia and in the world. In this paper, the authors would translate it into English and give their analysis at the end of the paper, hoping that this could initiate more discussions in the English-speaking world.

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In 1996, Japan's Central Council for Education published its first report *The Model for Japanese Education in the Perspective of the 21st Century*. It states that helping children/students develop their zest for living is of utmost importance and so we should let them make use of most of their time in independent free trials and discoveries. In response to the statement, Japan's Curriculum Council suggested to introduce a new standardised lessoning system which requires students attending school lessons five days per week instead of the currently 5-6 days. The educational contents under the new system will also be carefully selected. Accordingly, in 1998, the Ministry of Education started to revise the school curriculum and the revised mathematics curriculum will be implemented in 2002. In the revised mathematics curriculum, students in each grade level are required to learn arithmetic. Apart from arithmetic, students of the 3rd to the 6th grade level are also required to learn a new subject, Integrated Learning. To assign some time for the learning of the new subject, time currently allocated to arithmetic is reduced. Table 1 shows the suggested number of class periods per year for the two subjects. Each class period is 45 minutes long. Lessons on each subject would run for at least 34 weeks in the 1st grade level and at least 35 weeks in other grade levels.

Table 1: Number of Class Periods of Arithmetic and Integrated Learning

Grade (age)	Arithmetic*	Integrated Learning	Total*
1st (6)	114 (22)	-	782 (68)
2nd (7)	155 (20)	-	840 (70)
3rd (8)	150 (20)	105	910 (70)
4th (9)	150 (20)	105	945 (70)
5th (10)	150 (20)	110	945 (70)
6th (11)	150 (20)	110	945 (70)

* The figures indicate the number of class periods of all the subjects and the figures in the parentheses indicate the number of class periods reduced.

The aims of Integrated Learning is to help students:

- (1) develop the abilities to find a task, learn, think, judge and solve problems by themselves;
- (2) grasp the method of learning and thinking, build up creativity and develop an ability to live independently.

ARITHMETIC: THE CURRICULUM

Overall Objectives

The aims of the arithmetic curriculum are to help students, through arithmetic activities about numbers, quantities and geometrical figures, acquire the fundamental understanding and skills of arithmetic, develop the abilities to logically consider their daily-life phenomena with insight, and cultivate the

attitude to find the pleasure of doing arithmetic activities, understand the true value of mathematical procedures, and willingly apply the understanding in their lives.

Objectives and Content in Each Grade

[1st grade]

Objectives

Through activities of manipulating concrete objects, help students:

- (1) Enrich their sense of numbers, understand the meaning and representation of numbers, and consider about addition and subtraction, their calculation and application.
- (2) Gain the requisite experience for the learning of quantities and measurements, and enrich their sense of magnitude of quantities.
- (3) Gain the requisite experiences for the learning of geometrical figures, and enrich their sense of these figures and space.

Content

A. Numbers and Calculations

Through activities such as counting the number of objects, help students:

- (1) Understand the meaning of numbers and be able to use numbers:
 - a. Compare the numbers of objects by manipulation such as correspondence;
 - b. Correctly count or represent the number and order of an object;
 - c. Make a sequence of numbers and represent them on a number line by considering their size and order;
 - d. Consider the relationship of a number to other numbers as a sum or difference between them;
 - e. Understand the meaning of numbers up to 100 and how to represent them.
- (2) Understand the meanings of addition and subtraction of numbers and **apply** these operations.:
 - a. Recognise the cases in which these operations may be applied, and consider how to represent them by mathematical expressions and interpret the expressions;
 - b. Understand how to calculate addition of 1-digit numbers and subtraction of them as inverse operation of addition, and be able to carry out the calculation;
- (3) Acquire the ability of counting concrete objects by grouping them together or dividing them into equal parts, and showing the results in a simple representation.

B. Quantities and Measurements

Through activities such as comparing lengths of objects, enrich students with **the** requisite experience for the learning of quantities and measurements:

- (1) Directly compare lengths of objects.
- (2) Compare lengths in terms of number of units by using any object nearby **and** taking the length of the object as one unit.

C. Geometrical Figures

Through activities such as observing and constructing solids, to enrich **students** with the

requisite experience for the learning of geometrical figures:

- (1) Recognise the shapes of objects or grasp their features.
- (2) Represent the position of an object by correctly using words of direction and position such as “front and rear”, “right and left” and “upward and downward”

【Terms / Symbols】

ones place, tens place, +, -, =

[2nd grade]

Objectives

Through activities such as treating concrete objects, help students:

- (1) Enrich students' sense of numbers, and deepen their understanding of the meaning and representation of numbers and the meanings and application of addition and subtraction. Also help them understand the meaning of multiplication and its calculation, and be able to carry out the calculation.
- (2) Help them understand the units of length and measurements, and enrich their sense of magnitude of quantities.
- (3) Help them gain more experience for the learning of geometrical figures, and enrich their sense of these figures.

Content

A. Numbers and Calculations

- (1) Enable students to understand the meaning of numbers and how to represent them, and to develop their abilities to use numbers:
 - a. Count objects by rearranging them into groups of the same size or by classifying them.
 - b. Understand how to represent numbers up to 4 digits by the decimal positional numeration system (Hindu-Arabic numeration system), and also the size and order of the numbers.
 - c. Understand the relative size of numbers by considering 10 or 100 as one unit.
 - d. Consider the relationship of a number to other numbers as a product of them.
 - e. Classify simple cases, and use numbers to represent them in the form of tables or graphs.
- (2) Deepen their understanding of addition and subtraction, and develop their abilities to use these operations:
 - a. Understand the relations between addition and subtraction.
 - b. Consider how to calculate addition and subtraction of numbers up to 2 digits, understand that these operations for 2-digit numbers are carried out by using the basic facts of the operations for 1-digit numbers, and be able to carry out the calculation. Furthermore, consider how to calculate them in column form.
 - c. Investigate the properties of addition and subtraction, and make good use of the properties for the purposes of calculation or checking the result.
- (3) Enable them to understand the meaning and application of multiplication:
 - a. Recognise the cases in which multiplication may be applied, and consider how to represent them by mathematical expressions and interpret the expressions.
 - b. Investigate the simple properties of multiplication and use the properties to

construct multiplication table and check the result of calculation.

- c. Understand multiplication table and correctly multiply 1-digit numbers by 1-digit numbers.

B. Quantities and Measurements

- (1) Enable students to understand the meaning of length and measure lengths of objects in simple cases:
 - a. Understand the meaning of the units of length and their measurement.
 - b. Consider about the different units of length (millimetre or mm, centimetre or cm, and metre or m).
- (2) Enable them to read a clock in daily life.

C. Geometrical Figures

Through activities such as observing and constructing the shapes of objects, enrich students' experience for the learning of geometrical figures:

- (1) Construct various shapes and decompose them.
- (2) Consider triangles and quadrilaterals, and be able to draw and construct these figures.

【Terms / Symbols】

unit, straight line, \times

[3rd grade]

Objectives

- (1) Deepen students' understanding of multiplication and help them make good use of addition, subtraction and multiplication. Furthermore, help them understand division, its calculation and application.
- (2) Help them understand the units of volume, weight and time, and their measurement.
- (3) Help them understand the basic geometrical figures by paying attention to their composing elements.
- (4) Enable them to arrange data and represent the data in tables and graphs, and help them understand the usefulness of tables and graphs.

Content

A. Numbers and Calculations

- (1) Deepen students' understanding of how to represent numbers, and develop their abilities to use numbers:
 - a. Consider the place of ten thousand (the Japanese character of "ten thousand" is "万" and its Japanese pronunciation is "man").
 - b. Consider the sizes of 10 times and 100 times of a whole numbers, and how to represent them.
 - c. Deepen their understanding of the relative size of numbers.
- (2) Enable them to carry out addition and subtraction, and develop their abilities to use these operations appropriately:
 - a. Consider about the addition and subtraction of 3-digit numbers, and understand that these operations are carried out by using the basic facts of the operations for 2-digit numbers.
 - b. Be able to carry out the calculation of addition and subtraction, and use the operations appropriately.

- c. Investigate the properties of addition and subtraction, and make good use of the properties for the purposes of calculation or checking the result.
- (3) Deepen their understanding of multiplication of whole numbers, make them capable of surely carrying out the calculation, and develop their abilities to use it appropriately:
 - a. Consider how to multiply 2-digit or 3-digit numbers by 1-digit numbers, as well as 2-digit numbers by 2-digits numbers, and understand that the calculation of multiplication is based on the multiplication table. Also understand multiplication in column form.
 - b. Be able to carry out the calculation of multiplication, and use the operation appropriately.
 - c. Investigate the properties of multiplication, and use the properties in calculation and checking the result.
- (4) Enable them to understand the meaning of division and how to use it:
 - a. Recognise the cases in which division may be applied, and consider how to represent them in mathematical expressions and interpret the expressions. Furthermore, understand the meaning of remainder of division.
 - b. Understand the relations between division and multiplication.
 - c. Be able to surely carry out the calculation of division where both the divisor and quotient are 1-digit numbers.
- (5) Make them consider how numbers are set on an abacus (the Japanese character of "abacus" is "算盤" and its Japanese pronunciation is "soroban") and how to use abacus to carry out simple addition and subtraction.
 - a. Consider how to represent numbers on an abacus.
 - b. Consider how to add and subtract with an abacus.

B. Quantities and Measurements

- (1) Make students understand length, volume and weight, and be able to measure them in simple cases:
 - a. Consider the unit of length (kilometre, or km).
 - b. Understand the meaning of the units of volume and weight and their measurement.
 - c. Consider the unit of volume (litre, or l).
 - d. Consider the unit of weight (gram, or g).
- (2) Enable them to estimate length and measure it using the appropriate units and instruments according to different purposes:
 - a. Consider the meaning of day, hour, minute and second, and understand their relationship.
 - b. Understand the calculation of time needed in simple cases in terms of number of hours.

C. Geometrical Figures

Through activities such as observing and constructing the shapes of objects, enable students to understand the basic geometrical figures:

- (1) Consider the composing elements of the geometrical figures by observing and constructing objects having the shape of a box.
- (2) Consider about squares, rectangles and right-angled triangles by paying attention to their composing elements, and be able to draw and form them, and to use these figures to cover a plane.

D. Quantitative Relations

Enable students to represent data in the form of tables and graphs, and to interpret them.

- (1) Classify and arrange data from simple viewpoints such as day, time or place, and represent them in a table.
- (2) Consider how to interpret and draw bar charts.

【Terms / Symbols】

sign of equality, right angle, \div

[4th grade]

Objectives

- (1) Deepen students' understanding of division, and help them use the operation appropriately. Also help them understand the meaning of addition and subtraction of decimal fractions and their calculation, and use them appropriately.
- (2) Introduce the meaning of area and how to measure areas of simple geometrical figures. Also help them understand the meaning of the magnitude of an angle.
- (3) Improve their understanding of the basic geometrical figures by paying attention to their composing elements.
- (4) Show them the method to represent quantities and interpret their relations by using mathematical expressions or graphs, and help them investigate the relations between different expressions and consider how to classify and arrange them according to their nature.

Content

A. Numbers and Calculations

- (1) Enable students to represent whole numbers by the decimal positional numeration system:
 - a. Consider about the units such as hundred million (the Japanese character of "hundred million" is "億" and its Japanese pronunciation is "oku"), trillion (equal to billion in the U.K.; the Japanese character of "trillion" is "兆" and its Japanese pronunciation is "choh") and have an overview of the decimal positional numeration system.
 - b. Consider the sizes of 10 times, 100 times of a whole number and how to represent them.
- (2) Introduce them with the knowledge of round numbers and use rounding according to different purposes:
 - a. Recognise the cases in which round numbers may be applied.
 - b. Understand about rounding or counting fractions of 0.5 or above as one unit and disregarding fractions of 0.4 or below.
- (3) Improve their understanding of division of whole numbers, and enable them to calculate accurately. Also develop their abilities to use division appropriately:
 - a. Consider how to divide 2-digit or 3-digit dividend by 1-digit divisor, and understand that the calculation of this kind of division is based on the operation where both the dividend and divisor are 1-digit numbers. Also understand how to carry out division in column form.
 - b. Be able to carry out the calculation of division, and use the operation appropriately.
 - c. Investigate the relations between dividend, divisor, quotient and remainder, and summarise it in the following expression:

$$(\text{dividend}) = (\text{divisor}) \times (\text{quotient}) + (\text{remainder})$$

- d. Investigate the properties of division and use the properties in calculation and checking the result.
- (4) Make them understand the meaning of and the way to represent decimals, and the meaning of addition and subtraction of decimals and their calculations:
 - a. Use decimals to represent the size of fractional parts, and consider the notations of decimals and the position of $1/10$ (one-tenth).
 - b. Understand that the system of representing decimals is the same as that of representing whole numbers, and also understand the relative size of decimal numbers.
 - c. Consider how to calculate addition and subtraction of decimals up to $1/10$ (one-tenth) and be able to carry out them.
- (5) Help them understand the meaning and representation of fractions:
 - a. Use fractions to represent the size of fractional parts or the size of equal parts by division, and consider how to represent fractions.
 - b. Understand that a fraction is represented as the sum of some unit fractions.

B. Quantities and Measurements

- (1) Understand the meaning of area and its measurement in simple cases:
 - a. Understand the meaning of the unit of area and its measurement.
 - b. Consider about the unit (square centimetre or cm^2).
 - c. Consider how to measure the areas of squares and rectangles, and be able to apply the methods.
- (2) Understand the magnitude of angle and its measurement:
 - a. Grasp the meaning of the magnitude of an angle as the size of rotation, and the meaning of the unit and its measurement.
 - b. Consider about the unit of the magnitude of angle (degree or $^\circ$).

C. Geometrical Figures

Through activities such as observing and constructing geometrical figures, deepen students' understanding of the basic geometrical figures:

- (1) Consider about isosceles and equilateral triangles by paying attention to their composing elements, and be able to draw and form these triangles, and ⁽⁺⁾ to cover a plane with them.
- (2) Consider about angles in relation to the basic geometrical figures.
- (3) Consider about the centre, diameter and radius of a circle, and be able to draw and form a circle. Furthermore, to consider about a sphere and its diameter etc. in relation to a circle.

D. Quantitative Relations

- (1) Enable students to represent and investigate the relations between two quantities which vary in company with each other:
 - a. Investigate the quantitative relations in simple cases by considering the corresponding quantities or by representing the ordered pairs of corresponding values in a table.

- b. Represent the state of two varying quantities in a broken-line graph and interpret the features of their variation.
- (2) Enable them to concisely represent quantitative relations by mathematical expressions and interpret these expressions:
 - a. Understand mathematical expressions using some of the four operations and the expressions using parentheses i.e., (), and correctly calculate them.
 - b. Understand the idea about formula and apply formula.
- (3) Enable them to gather, classify and arrange data according to different purposes and investigate their features:
 - a. Investigate possible cases about two cases.
 - b. Check up on the lack or duplication of data.
 - c. Represent the data in broken-line graphs, and investigate the features and tendencies from these graphs.

【Terms / Symbols】

sum, difference, product, quotient, whole number, number line, decimal point, denominator, numerator, mixed fraction, proper fraction, improper fraction.

[5th grade]

Objectives

- (1) Improve students' understanding of the meanings and representation of decimals and fractions. Help them understand the meanings of multiplication and division of decimals, their calculation and application, and also understand the meanings of addition and subtraction of fractions, their calculation and application.
- (2) Enrich their understanding of the method to measure area, and further to measure the area of the basic plane geometrical figures.
- (3) Help them understand geometrical figures by paying attention to their composing elements and positional relations, and deepen their understanding of these figures.
- (4) Help them use percentage graph or circle graph to handle statistical data, and represent quantitative relations in mathematical expressions, interpret these expressions, and investigate their relations.

Content

A. Numbers and Calculations

- (1) Deepen students' understanding of the properties of whole numbers, which are represented by the decimal positional numeration system:
 - a. Consider that whole numbers may be classified into odd numbers and even numbers.
 - b. Consider about divisors and multiples.
- (2) Enrich their understanding of whole numbers and decimals from the idea of numeration system, and efficiently use this idea in calculation:
 - a. Make the multiples of a number by multiplying that number by 10, 100, $\frac{1}{10}$, $\frac{1}{100}$, and investigate their relations.
- (3) Make them consider about the meanings of multiplication and division in decimals, and apply the operations appropriately.
 - a. Understand the meanings of multiplication and division in the cases where the multiplier and divisor are whole numbers.
 - b. From the idea of multiplication and division, understand the meanings of

multiplication and division in the cases where the multiplier and divisor are decimals.

- c. Consider how to calculate multiplication and division in decimals, and be able to carry out them.
- (4) Deepen their understanding of fractions and help them understand the meanings of addition and subtraction in fractions with a common denominator, and apply the operations appropriately.
 - a. Use simple cases to make them aware that there are fractions of the equal size.
 - b. Transform whole numbers and decimals into fractions, and represent fractions as decimals.
 - c. Consider that the result of division of whole numbers can be always represented as a single number by using fractions.
 - d. Consider how to add and subtract in fractions with a common denominator, and be able to carry out them.
- (5) Deepen their understanding of rounding numbers:
 - a. Estimate the sum and difference of rounded numbers according to different purposes.

B. Quantities and Measurements

Deepen students' understanding that the area of the basic plane geometrical figures may be found by calculation and guide them to find the area:

- (1) Consider how to find the area of a triangle or a parallelogram, and apply the method.
- (2) Consider how to find the area of a circle, and apply the method.

C. Geometrical Figures

Through activities such as observing and constructing geometrical figures, deepen students' understanding of the basic plane geometrical figures and make them pay attention to their composing elements and positional relations.

- (1) Understand the relations of lines such as parallelism and perpendicularity.
- (2) Consider about parallelograms, trapeziums and rhombuses, be able to draw and form them, and to cover the plane in the form of tessellation.
- (3) Find the simple properties of the basic geometrical figures, and use these properties to investigate and construct the figures.
- (4) Understand the meaning of circle ratio.

D. Quantitative Relations

Enable students to summarise the properties of the four fundamental operations:

- (1) Deepen their understanding of the commutative law, associative law and distributive law.
- (2) Help them understand the meaning of percentage and apply it.
- (3) Enable them to classify and arrange data according to different purposes, and to represent data by circular graphs and band graphs.
- (4) Deepen their understanding of the way of viewing or investigating the quantitative relations of quantities represented by simple expressions by paying attention to the correspondence between the quantities.

【Terms / symbols】

parallel, perpendicular, diagonal line, %.

【6th grade】

Objectives

- (1) Deepen students' understanding of addition and subtraction in fractions and enable them to appropriately use them. Also help them understand the meanings and representation of multiplication and division in fractions, their calculation and application.
- (2) Help them understand the meaning of volume, and measure the volume of simple geometrical figures; furthermore, help them understand the meaning of speed and measure the value of speed.
- (3) Make them pay attention to the composing elements and positional relations of the basic solid figures, and deepen their understanding of them.
- (4) Enable them to understand the meaning of ratio and proportion, and consider quantitative relations by the idea of function.

Content

A. Numbers and Calculations

- (1) Deepen their understanding of the properties of whole numbers.
 - a. Consider about divisors and multiples.
- (2) Deepen their understanding of fractions, help them understand the meaning of addition and subtraction in fractions with different denominators, and make them capable of applying them appropriately.
 - a. Understand that the size of a fraction whose numerator and denominator are multiplied or divided by the same number is equal to the original size.
 - b. Consider a fraction as equal to, larger or smaller than another fraction in size, and summarise the methods of comparing the size of fractions.
 - c. Consider how to calculate addition and subtraction in fractions with different denominators, and be able to carry out the calculation.
- (3) Help them understand the meanings of multiplication and division in fractions, and apply the operations appropriately.
 - a. Understand the meanings of multiplication and division in the cases where the multiplier and divisor are whole numbers.
 - b. From the idea of calculating fractions in the cases where the multiplier and divisor are whole numbers or decimals, understand the meaning of multiplication and division in the cases where the multiplier and divisor are fractions.
 - c. Consider how to calculate addition and subtraction in fractions, and be able to carry out the calculations.
- (4) Deepen their understanding of round numbers:
 - a. Estimate the size of a product or a quotient by using round numbers according to different purposes.

B. Quantities and Measurements

- (1) Enable students to grasp the approximate shape of a figure and to measure its approximate area.
- (2) Help them understand the meaning of volume, and be able to measure the volume in simple cases:
 - a. Understand the units of volume and their measurement.
 - b. Consider about the units (cubic centimetre or cm^3 , and cubic metre or m^3) of volume.
 - c. Consider how to measure volume of a cubic or a rectangular parallelepiped, and be

able to apply the method.

- (3) Make them consider how to compare and represent a quantity which may be regarded as the ratio of two different kinds of quantities, and be able to apply the method:
 - a. Use the idea of “per unit”.
 - b. Understand the meaning of speed and its representation, and consider how to calculate and find speed.

C. Geometrical Figures

Through activities such as observing and constructing geometrical figures, deepen students' understanding of the basic geometrical solid figures and make them pay attention to the composing elements and positional relations of the figures.

- (1) Understand a cube and a rectangular parallelepiped.
- (2) Understand parallelism and perpendicularity of lines and planes about rectangular parallelepiped.
- (3) Consider about angular prisms such as prisms and quadrangular prisms, and circular cylinders.

D. Quantitative Relations

- (1) Enable students to understand the meaning of ratio in simple cases.
- (2) Help them develop the ability to consider the relations between two quantities which vary in company with each other:
 - a. Understand the meaning of direct proportion and investigate its features in simple cases by using tables and graphs.
- (3) Enable them to understand the meaning of average and apply it.

【Terms / Symbols】

greatest common divisor, least common multiple, reduction, reduction to a common denominator, plane, base, side, face, ∴

REMARKS ON THE COURSE OF STUDY

As to the content of curriculum, the following points should be noted:

- (1) At the fourth grade or later, teachers should help students use “soroban (Japanese abacus)” or hand-held calculators adequately.
- (2) Teachers should pay more attention to enriching students' sense of quantities and geometrical figures, and improving their abilities to apply tables or graphs.

Contents moved from some grades to the other:

A Numbers and Calculations

2nd grade → L.S.S. (lower Secondary School): expressions in inequality ($<$, $>$)

B Quantities and Measurements

6th grade → L.S.S.: the surface areas and volumes of cones and cylinders

C Geometrical Figures

5th grade → L.S.S.: congruence of geometrical figures, regular polygon, sector

6th grade → L.S.S.: line symmetry, point symmetry, reduced figure, extended figure, pyramid, circular cone, development figure of cylinders

D Quantitative Relations

5th grade → L.S.S.: expression in the letters such as a, x etc.

6th grade → L.S.S.: expression of direct and inverse proportions, and how to investigate the possible cases

Contents cut off in some grades

A Numbers and Calculations

3rd grade: addition and subtraction of 4-digit numbers, multiplication of 3-digit number by 2-digit number.

4th grade: multiplication of 3-digit number by 3-digit number, division of 3-digit divisor, [addition, subtraction, multiplication and division] in decimals (up to the second decimal point)

5th grade: addition and subtraction containing mixed fractions

6th grade: multiplication and division containing mixed fractions

B Quantities and Measurements

4th grade: units of area a, ha

4th grade: exchange among units in complicated cases

5th grade: area of trapezium, area of polygon, capacity

6th grade: efficient measurement by using direct proportion etc., metric system and relations among their units, and efficient use of them

C Geometrical Figures

4th grade: the mutual relations among quadrangles, the way to represent the position of an object in space

5th grade: regular polygon

D Quantitative Relations

6th grade: value of ratio, frequency distribution, devising useful tables and graphs, dispersion of data, ratio of statistical data and tendency of a population