Teaching and learning mathematics with understanding: the case of converting denary numbers into binary numbers

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It is commonly agreed that learning mathematics with understanding is more desirable than learning by rote. However, we may not have a clear conception of what constitutes mathematical understanding. The purpose of this article is to make use of a framework of mathematical understanding suggested by Hiebert and Carpenter (1992) to discuss how the conversion of denary numbers into binary numbers should be taught so that students' understanding of mathematics can be promoted.

Hiebert and Carpenter suggest to use this metaphor to describe mathematical understanding: inside our internal network of knowledge, the junction, or nodes, can be thought as pieces of represented information, and the threads between them as the connection or relationship. Understanding is defined in terms of the way information is represented and structured: *a mathematical idea or procedure or fact is understood if it is part of an internal network, and* <u>*the degree of understanding*</u> *is determined by the number and the strength of the connection. When a student learns a piece of mathematical knowledge without connecting it with items in his or her existing networks of internal knowledge, he or she is learning without understanding.*

Converting denary numbers into binary

Let us examine the way that secondary one students in Hong Kong are taught to convert denary numbers into binary numbers. Usually the procedure is given without explanation:

(B) FROM DENARY TO BINARY

Example 6 Change the following denary numbers into binary numbers:

(a) $35_{(10)}$ (b) $213_{(10)}$

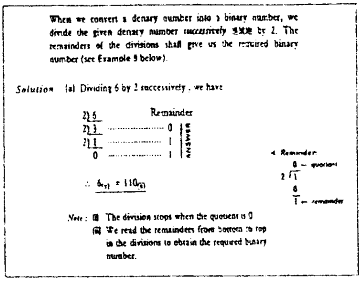
Solution (a)

2 | 35
2 | 17 .... remainder 1
2 | 8 .... remainder 1
2 | 4 .... remainder 0
2 | 2 .... remainder 0
2 | 1 .... remainder 0
0 .... remainder 1

∴ $35_{(10)} = \underline{100011_{(2)}}$ (from the remainders) *Ans.*

(Manhattan Book 1, 1988, P.13)

Even when there is explanation, the focus is on the procedure itself, not on the underlying principle of the procedure:


When we convert a denary number into a binary number, we divide the given denary number *successively* 逐次 by 2. The remainders of the divisions shall give us the required binary number (see Example 5 below).

Solution (a) Dividing 6 by 2 successively, we have

	Remainder
2) 6	
2) 3	0
2) 1	1
0	1

∴ $6_{(10)} = 110_{(2)}$

Note: (i) The division stops when the quotient is 0
(ii) We read the remainders from bottom to top in the divisions to obtain the required binary number.

(Canotta Book 1A, 1994, P.53)

Although most students can follow the suggested procedure without difficulty, the following questions may arise:

"Why should we use division?"

"Why should we read the answer from the bottom to the top?"

"What does each remainder represent?"

If students master the procedure without connecting it to their exiting knowledge of the binary and denary numeral systems, they are learning the procedure with little understanding. In order to remedy the deficiency of the textbook, an alternative approach is required.

An alternative approach

In converting denary numbers into binary numbers, students should appreciate that:

(1) the place values are changed from ones, tens, hundreds, ... to ones, twos, fours, eighties, ...

(2) only the digits 0 and 1 are allowed to use. Without this rule, a denary number can be represented in many different ways.

Take the number 16 as an example, the following are <u>some</u> of the possible ways of representing it in the binary system:

(a)

			8	0
(16)	(8)	(4)	(2)	(1)

(b)

	2	0	0	0
(16)	(8)	(4)	(2)	(1)

(c)

1	0	0	0	0
(16)	(8)	(4)	(2)	(1)

Similarly, the number "13" can be represented as:

			6	1
(16)	(8)	(4)	(2)	(1)

Or:

		3	0	1
(16)	(8)	(4)	(2)	(1)

Or:

		1	4	1
(16)	(8)	(4)	(2)	(1)

Or:

	1	0	2	1
(16)	(8)	(4)	(2)	(1)

Or:

	1	1	0	1
(16)	(8)	(4)	(2)	(1)

However, 10000 and 1101 are the only acceptable forms for 16 and 13 respectively in the binary system since the rest contain digits other than 0 and 1

A smaller digit in a place with greater place value can be the same representation as a bigger digit in a place with smaller place value. In converting a denary number into a binary one, we can put the number in the first place of the binary number, and make use of division to keep on re-grouping the number. This procedure ends until no digits other than 0 and 1 appear in any of the places. For example:

25 (base ten)

=

12	1
(2)	(1)

(25÷2 = 12, remainder = 1)

=

6	0	1
(4)	(2)	(1)

(12÷2 = 6, remainder = 0)

=

3	0	0	1
(8)	(4)	(2)	(1)

(6÷2 = 3, remainder = 0)

=

1	1	0	0	1
(16)	(8)	(4)	(2)	(1)

(3÷2 = 1, remainder = 1)

The above procedure is similar to the textbook approach. (However, in this procedure, we need not stop until the dividend becomes zero.) Although it is less efficient, it illustrates why we could use division to convert denary numbers into binary numbers. It also demonstrates the importance of the remainders and why they should be read upward to obtain the answer.

This teaching approach may not be appropriate for all students. For some students, we could simply employ the textbook approach and ask them to explain why it works. The most important thing is to help students connect the procedure with their existing mathematical knowledge.

The subtraction algorithm

Instead of using the division algorithm, we can use a subtraction algorithm to convert denary numbers into binary numbers. For example:

$25_{\text{(base ten)}}$

=	1	0	0	0	0	plus 9	(25-16=9)
	(16)	(8)	(4)	(2)	(1)		
=	1	1	0	0	0	plus 1	(9-8=1)
	(16)	(8)	(4)	(2)	(1)		
=	1	1	0	0	1		
	(16)	(8)	(4)	(2)	(1)		

This procedure is closely related to the procedure of converting binary numbers into denary numbers.

Conclusion

In converting denary numbers into binary, many textbooks fail to point out the importance of the concept of place and place value. The concept that limited number of digits are allowed to use in a particular number system is also neglected. Without connecting the above concepts to the standard procedure, many students would learn the procedure by rote, without understanding the underlying principle. Being mathematics teachers who want to promote learning with understanding, we have to design appropriate teaching strategies to remedy the deficiencies of the textbooks when they fail to connect the new knowledge item with students' existing knowledge of mathematics.

Reference: Hiebert, J. & Carpenter, T.P.(1992). Learning and Teaching with Understanding. In D.A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. A Project of the National Council of Teachers of Mathematics. New York: Macmillan.