Good Mathematics Teaching: A Student-teacher's Perspective

Paul Sze
Department of Curriculum and Instruction
The Chinese University of Hong Kong

I was having an unstructured interview with Denny (a pseudonym), a student-teacher majoring in mathematics teaching, for a case study which aimed to probe the concerns, frustrations, aspirations, etc., that student-teachers experience as they embark on the Teaching Practice. During this interview, Denny happened to bring up the issue of good mathematics teaching, and I prompted him to clarify what he meant by teaching mathematics well. This resulted in an extended effort on Denny's part to define good mathematics teaching. While reading the transcript of this interview afterwards, it occurred to me that this section of the transcript, where Denny tried to define good mathematics teaching, might be of interest to those concerned with teacher preparation in mathematics education.

This paper presents that section of the transcript (permission has been given by Denny to reproduce the data here) as an example of how one student-teacher might conceptualize good mathematics teaching. I shall begin with an introduction to the background of the interview. This will be followed by the extract of the transcript itself. Finally, I shall attempt to interpret the data briefly. This article should not be read as a research report, and in fact, mathematics education is not my specialty. Readers who are interested or involved in teacher education in mathematics teaching are invited to make their own interpretations of the data, which is my ultimate purpose in presenting the data here.

The original case study: TP and the student-teacher

The transcript reproduced below is part of the data collected from a case study which aimed to collect thick descriptions of how one student-teacher experienced the Teaching Practice (TP), with special attention to his concerns, worries, frustrations, expectations, etc., about the TP. Data

collection for this case study consisted of (1) unstructured interviews before, during, and after TP, and (2) the keeping of a diary by the student-teacher during the four weeks of TP.

Two unstructured interviews were carried out before the TP with Denny, the participant in the case study. The first one was carried out three weeks before the TP, when Denny had just learnt about his TP school assignment. The second interview took place four days prior to the TP, when Denny was busy preparing for his teaching The transcript reproduced below comes from the second unstructured interview. This interview lasted for about forty-five minutes, and was carried out in Cantonese.

The student teacher, Denny, was a student-teacher on the one-year full-time Postgraduate Diploma in Education (PGDE) programme offered by the Chinese University of Hong Kong. Denny was a Mathematics major in his undergraduate studies, and he also majored in the teaching of Mathematics in the pre-service teacher preparation programme.

I came to be acquainted with Denny through the student research assistants scheme at the university. My own area is second language teacher education but teacher thinking is also my research interest. Although I had been seeing Denny periodically, I was not directly involved in his studies in the PGDE programme. When I asked Denny whether he would like to help me with the case study, he readily agreed.

The transcript

The transcript reproduced below comes from the second interview. Ten minutes into this interview, Denny happened to mention that he was concerned about teaching well during the TP. I then prompted him to tell me more about his conception of good mathematics teaching. Upon my prompting, Denny then spent the next ten minutes trying to explain his idea of teaching mathematics well. The whole interview took place in Cantonese, and was tape-recorded. Afterwards, the recorded interview was translated and transcribed. The extract reproduced below is a verbatim record of that part of the interview. In the extract, "I" stands for the investigator, that is myself, and "D" stands for Denny, the student-teacher.

Conceptualizing good mathematics teaching

I: Does it mean that at this stage, four days before the TP, you're more concerned about whether you can teach well than whether you can teach something more difficult as your students are Band 1 students?

D: I think so. You have to actually see how they perform before you can decide whether to teach something more difficult. Well, students are different. Some students are more interested in Maths - maybe I can give something more to them. Some people may think, well it's good enough if I can do the basic things. Every student is different. But I think you have to do the basic things well first. Firstly, you have to teach well. Secondly, it depends on the situation whether I can teach something harder, because when you teach a large class, you may be able to raise one or two challenging issues, but you may not really be able to go deep into the issues.

I: How would you define "teaching well"? For instance, under what circumstances would you be assured that you have taught well? What is your conception of teaching well?

D: To me, teaching well is not just that they can work out the maths problems. They should at least be able to develop some maths concept; they may be able to solve some maths problems on paper correctly, but they may not really understand what it all means, they should at least know what this particular piece of mathematics is about. This is more important that whether they can work out the answers. So I would emphasize two things: first whether they can solve the problem; second whether they understand what is happening.

I: What do you mean by "understanding what is happening"?

D: They may be following some steps in solving a problem in a mechanical way. But I want them to know that when they go from one step to the next, the process is a thinking process - it's a mathematical thinking process. What does that mean? Of course logic and reasoning are important in maths. But in solving a problem, we're transforming a problem into mathematical thinking. A mathematical concept is involved. How can we start with a mathematical concept in dealing with a problem? Knowing what is happening means making the connection. Whether they can display the steps on paper is not the real issue.

I: From the topics you have to teach in the coming TP, or the topics from

any junior maths coursebook, can you give an example to illustrate what you mean by "understanding what's happening"?

D: Let me try to think of a topic. One of my topics in F.1 is to prove that two triangles are congruent ... To prove that, you need at least three conditions. These three conditions contain at least one side. As a basic approach, you need 3 statements, then you draw a conclusion. Maybe some students have difficulty with this ... for example in this example (drawing on paper), you have a case of SAS, you have 3 obvious conditions. But some students may not understand how come when you have SAS, you can prove that 2 triangles are congruent ...The mathematical thinking is that in proving congruency you need a minimum number of sufficient conditions. Of course there are other conditions, and other non-conditions. I want them to be clear about this. ...

I: Could you clarify this again? As far as this example is concerned, what would your students need to know or demonstrate, before you can safely conclude that they "know what's going on"?

D: In this example, it's difficult to tell whether they can solve the problem only, or whether they also understand what's going on. This is the most difficult part. A moment ago, an idea crossed my mind - I could add a distracter to the problem. If a student really understands - for example if I provide one more distracter condition - then why these particular three conditions, why not that one? Then under the circumstance, they know that they have to choose the right conditions. What I meant just now is, very often, these problems are too easy to solve, because there're too few conditions. Students can solve them with their eyes closed. It's not difficult to arrive at the answer. If they make a mistake, maybe it's just a question of presentation. A student may work out the answer mechanically. But maybe mathematical understanding is not something we can see by looking at the answer to a problem. It's a whole process.

Just now, I remembered that to prove congruency, you only need to find 2 equal angles. The third angle will then always be equal. Some books still rely on three conditions. But some books are changing ...two angles is enough. This involves some mathematical thinking. This example may say something about the process. But still it's difficult to tell whether a student can only do it, or whether he is doing it with mathematical understanding. If I give them an exercise

and they can do all the problems. I can only tell whether they can do the problems or not - I still can't tell whether mathematically they've made any progress.

I: Do you mean in your teaching you're more concerned with some kind of mathematical thinking. Proving congruency is just a process?

D: Yes, it's just a process. What's important is what they can learn in the process of solving problems, not what they can write out. Of course it's important that they can work out the answers, for example it's important that they don't apply formulae wrongly. But I'm more concerned with – you know as they progress to upper forms, how they think is important, especially when they go up to F.6 and F.7.

A tentative analysis

Here I would like to offer a tentative analysis of Denny's conception of good mathematics teaching.

Denny was concerned with teaching well. To him, mathematics teaching was not simply discharging certain instructional duties in relation to the subject-matter of mathematics. Denny hoped to be teaching maths well.

For Denny, good mathematics teaching was not tied to the subject-matter in hand. In fact, the subject-matter of mathematics was only a vehicle within a long-term effort to accomplish something else which was more important and valuable. That something else was mathematical thinking and understanding. In comparison, the ability to solve the mathematical problems in hand was of secondary importance. Denny would not be contented if getting at the right answers was the only thing that his students could do.

Moreover, Denny's conception of good maths teaching was learner-oriented. It was the learner's understanding that ultimately defined good teaching. Throughout his attempt to define good mathematics teaching, Denny rarely referred to teachers' classroom instructional skills as an indicator of good mathematics teaching.

Good mathematics teaching, therefore, was a long-term effort, whose result might not be seen at the end of individual lessons, or simply by checking the answers produced by students.

Denny's process of defining good mathematics teaching might also

provide interesting insights for researchers into a student-teacher's schemata for understanding mathematics teaching. Denny's concept of good mathematics teaching did not seem to be a well-defined one. He had a general notion that good mathematics teaching was more than making sure that students obtained the right answers, but his conceptualization of good mathematics teaching appeared to be lacking in precision and plurality of perspectives. Denny repeatedly resorted to expressions such as "some kind of mathematical thinking", "mathematical understanding", and "knowing what's going on". He conceded time and again that it was difficult to tell whether mathematical thinking/understanding had taken place or not.

In search of further interpretations

This effort by Denny to define good mathematics teaching shows that even as a student-teacher who was about to embark on TP, good teaching to Denny was more than successful survival in the classroom. Good mathematics teaching consisted of developing students' mathematical thinking and understanding, although Denny was not able to further describe what mathematical thinking and understanding constituted. This agrees with the finding by some researchers that novice teachers, as opposed to expert teachers, employ a much simpler schemata when thinking about teaching problems.

Finally, it has to be pointed out that this brief analysis may have been influenced by a number of factors, which are inherent in interpretive research. For example, the fact that I am not a subject specialist in mathematics and mathematics education may have affected the discourse that Denny had resorted to in explaining to me his conception of good mathematics teaching. My relationship to him is another issue to consider. Also, the interview as a social act may have constrained the data I was able to obtain.