# A Reflection on Teaching a Probability Problem

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## 1. Introduction

In a lesson of S5 Mathematics, I posted a problem about probability (see section 2) so as to check whether my students could master the skill of considering complementary events. It turns out that some of them found it difficult to handle. I then outlined two possible methods to solve it. Later on, one student put down her solution on a piece of paper and asked me why it is incorrect. I guess that it should be a common mistake and so I tried to explain the details to her. The details of the lesson are shown in the next section.

## 2. Details of the lesson

The problem that I asked my students to try is shown below.

- The problem:

Angel, Crystal and Dora are given some puzzles to play with. For any given puzzle, the probabilities that they are able to crack it are $\frac{3}{16}$ , $\frac{1}{36}$ and $\frac{6}{13}$ respectively. Assuming that they deal with the puzzle independently and each of them is given two puzzles to play with, find the probability that at least one of them cracked only one puzzle.

- Suggested solution:

**(Method 1)**

Required probability

$= 1 -$ P(none of them cracked only one puzzle)

$= 1 -$ P(Angel cracked 0 puzzle or 2 puzzles) ×
P(Crystal cracked 0 puzzle or 2 puzzles) ×
P(Dora cracked 0 puzzle or 2 puzzles)

$$= 1 - \left[\left(1-\frac{3}{16}\right)^2+\left(\frac{3}{16}\right)^2\right]\times\left[\left(1-\frac{1}{36}\right)^2+\left(\frac{1}{36}\right)^2\right]\times\left[\left(1-\frac{6}{13}\right)^2+\left(\frac{6}{13}\right)^2\right]$$

$\approx 0.699.$

**(Method 2)**

Required probability

= 1 – P(none of them cracked only one puzzle)

= 1 – P(Angel did not solve only 1 puzzle) $\times$ P(Crystal did not solve only 1 puzzle) $\times$ P(Dora did not solve only 1 puzzle)

$$= 1 - \left[1-\left(\frac{3}{16}\right)\left(1-\frac{3}{16}\right)\times 2\right]\times\left[1-\left(\frac{1}{36}\right)\left(1-\frac{1}{36}\right)\times 2\right]\times\left[1-\left(\frac{6}{13}\right)\left(1-\frac{6}{13}\right)\times 2\right]$$

$\approx 0.699$.

## 3. Response from a student

After explaining methods 1 and 2 as shown in section 2, one student told me the following:

"Since the opposite of "cracked only 1 puzzle" is equivalent to saying that "either 0 puzzle cracked or 2 puzzles cracked", so the solution should be:

$$1-\left(\frac{13}{16}\times\frac{35}{36}\times\frac{7}{13}\right)^2-\left(\frac{3}{16}\times\frac{1}{36}\times\frac{6}{13}\right)^2 \text{."}$$

But then she was also wondering why she can't get the same numerical answer as what I did.

Clearly, she fell into the trap of missing out some cases. The main reason why she got it wrong is simply because she didn't view the situation globally. For instance, the case where Angel cracked 2 puzzles, Crystal cracked 0 puzzle and Dora cracked 0 puzzle should be ruled out too. In order to help her fix the bug, I demonstrated the third solution below:

**(Method 3)**

Refer to the Table 1 shown below. Each value in the grid represents the number of puzzles cracked by Angel, Crystal or Dora.

Table 1

| Angel | Crystal | Dora |
|---|---|---|
| 0 | 0 | 0 |
| 2 | 2 | 2 |
| 2 | 0 | 0 |
| 0 | 2 | 0 |
| 0 | 0 | 2 |
| 2 | 2 | 0 |
| 2 | 0 | 2 |
| 0 | 2 | 2 |

Thus, in order to meet the requirement of the story, we need to rule out all the 8 cases as depicted in Table 1. That is to say:

Required probability

$$= 1 - \left(\frac{13}{16}\times\frac{35}{36}\times\frac{7}{13}\right)^2 - \left(\frac{3}{16}\times\frac{1}{36}\times\frac{6}{13}\right)^2 - \left(\frac{3}{16}\times\frac{35}{36}\times\frac{7}{13}\right)^2$$
$$- \left(\frac{13}{16}\times\frac{1}{36}\times\frac{7}{13}\right)^2 - \left(\frac{13}{16}\times\frac{35}{36}\times\frac{6}{13}\right)^2 - \left(\frac{3}{16}\times\frac{1}{36}\times\frac{7}{13}\right)^2$$
$$- \left(\frac{3}{16}\times\frac{35}{36}\times\frac{6}{13}\right)^2 - \left(\frac{13}{16}\times\frac{1}{36}\times\frac{6}{13}\right)^2$$
$$\approx 0.699.$$

## 4. Reflection

In this paper, we suggested 3 possible solutions for solving a probability problem. All methods make use of the concept of considering complementary events. However, each method displays different features.

For method 1, students need only to transform the message of "not cracked only one puzzle" into "cracked 0 puzzle or cracked 2 puzzles". As compared with method 2, students have to apply the concept of complement twice! Worst still, when they consider the message of "cracked only one puzzle", they have to take care of the order. That's why students seldom do it by method 2 as it is harder. Lastly, method 3 is nothing new but it is tedious too as students have to list out all the cases one by one.

Fortunately, there were no students asking what if we need to solve it using direct method without the consideration of complement. If they did, we better give them a warning to do so as there are totally 19 cases (i.e. $3^3-8$) to be handled. In fact, teachers may also ask students to calculate the total number of cases for using direct method if the total number of puzzles available are 3 instead of 2 (i.e. $4^3-3^3=37$). In this way, students will get a deep feeling of the complexity using direct method.

## 5. Acknowledgement

I wholeheartedly thank my S5A students who are eager to try more challenging problems. Special thanks also go to Dr. Allen Leung and Dr. Tang, my M.Ed. lecturers at The Hong Kong Baptist University, who are always willing to give me encouragement and valuable advices.

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