A Note on Different Approaches to a Single Problem

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Different approaches to a single problem

It is believed that problem solving abilities can be enhanced if solutions to a single problem by different approaches can be demonstrated [Wong, 1987]. Indeed, many articles had demonstrated examples of mathematics problems for classroom discussion. For example, Wong N.Y. [Wong, 1992] demonstrated more than 10 approaches to prove the Arithmetic Mean - Geometric Mean inequality while Kwok, K.K. [Kwok, 1995] used 3 approaches to derive the quadratic formula and 6 methods to prove a special case of the Cauchy-Schwarz inequality.

A problem of quadratic equation

In the previous issue of *EduMath*, Mr. Mak K.W. presented three solutions to a typical area problem of quadratic equation [Mak, 2015]. The article is inspiring and the problem is interesting in itself.

In this article, we will present one more approach to the same problem and hope that it will arise more space for discussion.

The problem

Let us recall the statement of the problem first. In the diagram shown, *ABCD* is a square with side 20 cm long. Given that $CM = CK = x$ cm and the area of ΔMAK is 102 cm^2, find the value of x.

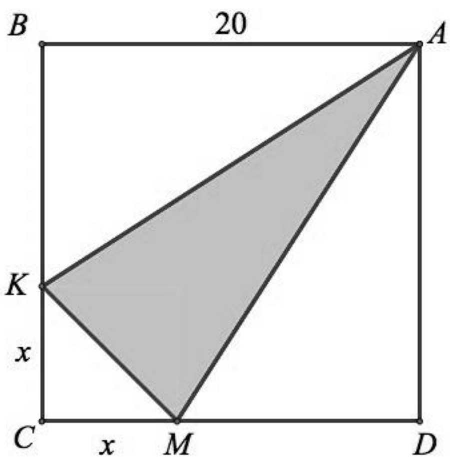

Figure 1

Mak [Mak, 2016] presented three solutions to the problem; all of them involved setting a polynomial equation of *x* to solve for *x*.

Yet another approach

We shall present a solution without solving any equation.

First, construct line segments *ME* and *KF* parallel to the sides of the square *ABCD* with *E* on *AB* and *F* on *AD* respectively. Suppose these two line segments intersect at *G* (see Figure 2(a)).

Now, since Δ*AKG* and Δ*BKG* share the same base KG and have the same height, they have the same area. Similarly, ΔA*GM* and Δ*DGM* also have the same area (see Figure 2(b)).

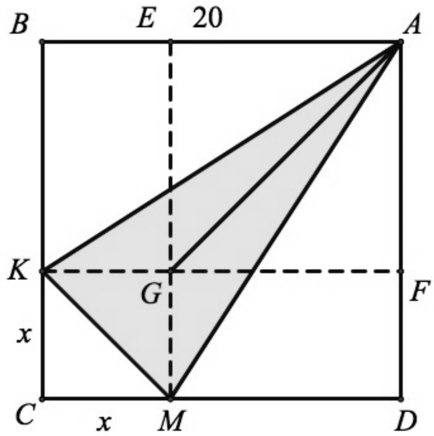

Figure 2(a)

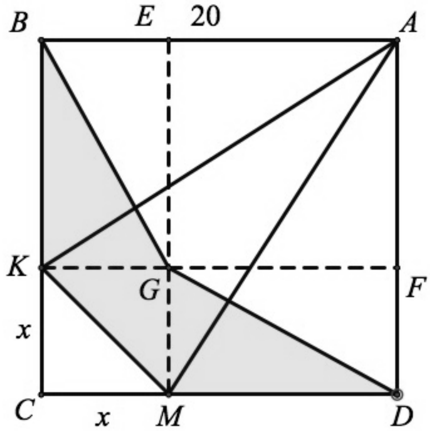

Figure 2(b)

Consequently, it is easy to see that if the area of ΔAKM is 102 cm^2, then the area of the L-shape *GEBCDF* is 204 cm^2 (see Figure 3).

Next, it follows that the area of the square *AEGF* is 20×20 cm^2 – 204 cm^2 = 196 cm^2 which implies that the side of the square *AEGF* is 14 cm and so $BK = EG$ = 14 cm. Finally we have $x = 20 - 14 = 6$.

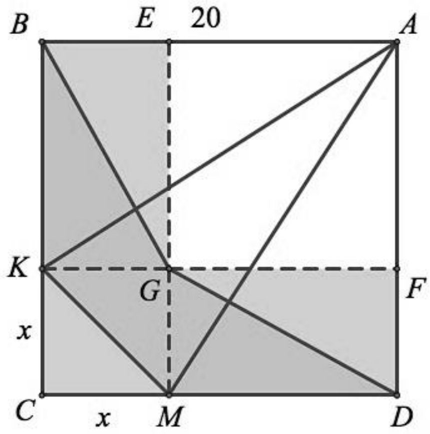

Figure 3

Note that this approach can be explained to students of lower form without the prerequisite of knowing the technique of solving quadratic equation.

Conclusion

"Looking Back" is an important process in mathematical problem solving [Pólya, 1962]. "Can you derive the result differently?" is the question proposed by Pólya for us during this process. Indeed, demonstration of different approaches will probably lead to a lot of class discussion. "Which method is most straightforward?", "Which method needs some clever trick?", "Which method can be useful to be generalize to other cases?". We believe that these discussions are worthwhile to students' learning process. Lastly, we would like to thank Mr. Mak for providing us such interesting problem for our discussion.

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