Let Students Talk in Mathematics Lesson

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1. Introduction

About mathematics learning, Jo Boaler has highlighted the role of talking over listening as, "… students often need to talk through methods to know whether they really understand them. Methods can seem to make sense when people hear them, by explaining them to someone else is the best way to know whether they are really understood" (Boaler, 2008, p.46). Most teachers are well aware of this but find it impossible to squeeze time to let their students talk. Moreover, examples of students' talk in lesson are rare. Due to their lack of exposure, teachers usually raise queries such as, "Is mathematics talk among students possible in a classroom lesson? What could they talk about? How would they talk? How could they learn by talking? How could my students talk when they don't have any common sense in mathematics?"

In order to encourage students to take up a proactive role in mathematics learning, our school has designed and taught a few lessons relying on students' talk. In such a way, we have also encountered the above queries in real situations rather than restricting ourselves in a self-fulfilling prophecy. This article is a reflection of one of those trial lessons.

2. Lessons designed

Two trial dialogic lessons (i.e. letting students learn via talking in the lessons) are about "surd" which is something new to Secondary 2 students.

In preparation meetings, teachers noted that students had been weak in understanding the meaning of square roots, particularly in surds. When the meaning of a surd is introduced with a formal definition, followed by drills in arithmetic or algebraic problems, the students usually cannot recognize surd as a numerical value like integer, fraction, or decimal. They will find it difficult in the cases below in their upper forms:

- Simplify $(\sqrt{a})^2$.
- Find the value of x in $2x+\sqrt{3}=4$.

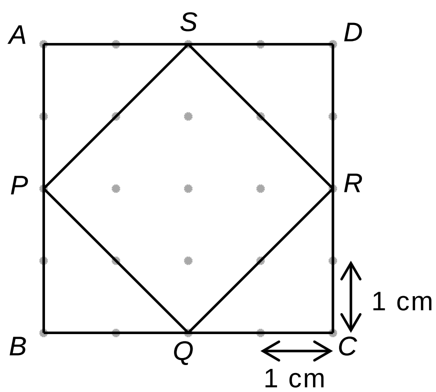
Therefore, lessons are designed to facilitate students in constructing a more practical meaning of a surd as the side length of a square. Activities are arranged to facilitate talking among students, where there could be meaning negotiation, answer justification, idea sharing or debate on alternative methods.

There are 4 major activities, which are,

Activity 1: The Starter — Teacher writes "$\sqrt{3}$" on the board and asks students the question, "What do you know about this?"

Activity 2:

In the figure, *ABCD* is a square of side 4 cm. *P*, *Q*, *R* and *S* are points on *AB*, *BC*, *CD* and *AD* respectively. Students are asked to use any of their pre-knowledge to find the area of the quadrilateral *PQRS*.

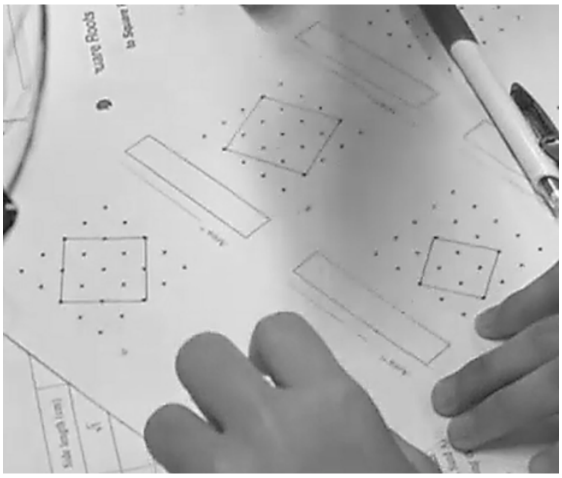


One significant of this activity is that the area of this tilted square is 8 cm^2 which is not a square number like 1, 4, 9, 16, …. This will lead to a practical meaning of $\sqrt{8}$ as the side of the tilted square. It is also designed as a preparation for the next activity.

Activity 3: How many squares can you find?

On the 5 × 5 dot paper provided, each dot is 1 cm from its nearest horizontal and vertical neighbors. On the dot paper, students need to

(i) draw squares of various sizes by connecting dots;
(ii) draw squares with as many different areas as possible;
(iii) label each square with its area.



This activity is designed to engage students in forming their own square shapes with areas in square numbers (e.g. 1, 4, 9, 16) and non-square numbers (e.g. 2, 5, 8, 10).

Activity 4: How would you estimate, the value of a surd, e.g. $\sqrt{8}$, without using a calculator?

During lessons, in order to encourage them to participate in the construction of abstract concepts, the students work in pairs, and more questions are asked to provide situations for them to think, to talk, and to make their own conclusions.

Therefore, if the lesson so designed is viewed as merely consisting of 4 activities to make learning interesting, the key objectives could be missed. There is far more to it. The activities are vehicles to help students build up their own ideas of what surds are through talking about their observation, intuition, memory and self-correction. In their talk during the activities, they also speculate about the possibilities, compare, negotiate and justify their ideas and undergo trial and error. These lead to a higher order of learning skills.

3. Students' talk

In Activities 2, 3 and 4 above, students might find it easier to talk about their answers, methods or rationale as they can treat them as a review of formulas or methods learnt before, such as the calculation of areas and the estimation of $\sqrt{8}$.

On the contrary, Activity 1 is more open, with students asked to tell what "$\sqrt{3}$" is. While some teachers may expect only a couple of standard answers from students, this time, in one of the classes that carried out the lesson as planned, no less than 12 different answers emerged from them. The teacher in that class tried to engage all students in various ways, and most of them did talk, enjoyed the participation and learnt well.

First, he arranged his 32 students into 16 pairs. The two members of each group were designated as A and B. They were invited to discuss with each other for 20 seconds about what "$\sqrt{3}$" is. Student A of each group wrote the group answer on the board. Their answers are shown below. (The numbering is added here for referencing.)

1. square root	5. the square root	9 the square of 3	13. square root
2. x^2	6. Find out the square root of 3	10 radical sign	14 square of 3
3. (1.732) 1.732050808	7. About Math's sign	11 1.732	15 factor
4. ? × ? = 3	8. square root	12 the square root of 3	16. $x \times x = 3$

Second, among all 16 written answers, though some of them are similar or even exactly the same, the teacher treated every answer as unique and insisted on asking every student B to explain A's written answer. She recognized that every answer made sense. These yielded fruitful student talk. For example, when student B of answer 3 was asked to explain, she demonstrated how she found 1.732050808 by pressing the particular keys on her calculator. When student B of answer 11 said that she found the value 1.732 from her calculator, the teacher further led her to qualify that the approximation was corrected to 3 decimal places.

In such a way, ideas in each student's mind were revealed. Moreover, everyone played a constructive part in the formation of meaning. They were the re-inventors and owners of the meanings, and their different facets and nuance.

Third, the teacher always gave positive feedback to encourage clarification and elaboration from the student herself. The teacher did not correct a student's answer directly. Students who had given incomplete answers e.g. "square of 3" (in answers 9 and 14) would make correction autonomously after listening to classmates' sharing.

Fourth, the teacher would lead students to complete their ideas according to their initial understanding, no matter how strange it seemed to be. For example, in answer 15, students showed a different facet of understanding. Student B said, "We want to write square root, but we have forgotten the right phrase, so we use the word, 'factor'." Then the teacher followed student's idea of treating $\sqrt{3}$ as a factor of 3 by asking for the other factor of 3. She furthermore helped that pair develop the idea of $\sqrt{3}\times\sqrt{3}=\left(\sqrt{3}\right)^2=3$.

Instead of only copying and memorizing the meaning given in the textbook, everyone managed to acquire richer and more significant meanings of the symbol '$\sqrt{3}$', through listening to classmates' explanations and trying to answer the teacher's questions.

In addition, through talking, and being respected in their talk, students got a new experience in learning mathematics. They were treated as one of the producers of new knowledge even if their initial idea could be incomplete or inaccurate. Chances had been provided for them to modify and refine their initial ideas. After Activity 1, students turned out to be more participative in Activities 2 to 4.

4. Reflection

The approach in this lesson was admittedly energy and time consuming, and so most teachers may not be able to do it all the time. But it's still worth it once in a while as it makes use of a strategy for deeper learning. In such a lesson, students are asked questions of what, how and why. Having thought through such things a few times, they may direct their thinking process towards them automatically, even in non-dialogic lessons.

References

Boaler, J (2008). *What's math got to do with it? Helping children learn to love their most hated subject — and why it's important for America.* New York: Penguin Group Inc.

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