From Inductive Exploration to Deductive Justification: The Discernment of Invariants in Pre-constructed Dynamic Geometry Sketches

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Dynamic geometry software (such as Geogebra) is a powerful tool for students to investigate geometric relation through the dynamic sketches, and help the students to acquire geometrical insights and knowledge. However lots of the researches point out that the passage from "exploratory" geometry in dynamic geometry environment (DGE) to the deductive Euclidean geometry is neither simple nor spontaneous. The problem of combining inductive exploration with the deductive structure of geometrical proofs has been the subject of a number of research studies (Mariotti, 2001, 2006). On the other hand, Christou et al. (2004) highlight that dynamic geometry may be useful in helping students understand problems in geometry but it does not contribute to the development of their abilities in proofs. The nature of DGE is inductive, and it leads to an experimental-theoretical gap between the dynamic geometry sketch and the formal deductive proof.

Is the bridging of the gap possible? Leung (2003, 2007, 2008, 2009) suggests that the discernment of invariants can facilitate students to have a deeper understanding of deductive geometry in DGE. When students perform dragging in dynamic sketches, some lengths and angles will change (vary) in magnitude but some may not. Meanwhile the lengths, angles, etc. may not change throughout dragging, e.g. dragging the vertices of a pre-constructed isosceles triangles will not change its property (i.e. base angles are equal in magnitude). Such unchanged geometric properties are called invariants. Actually discernment of invariants in DGE can facilitate students to proceed deductive justification of theorems. The following is a demonstration:

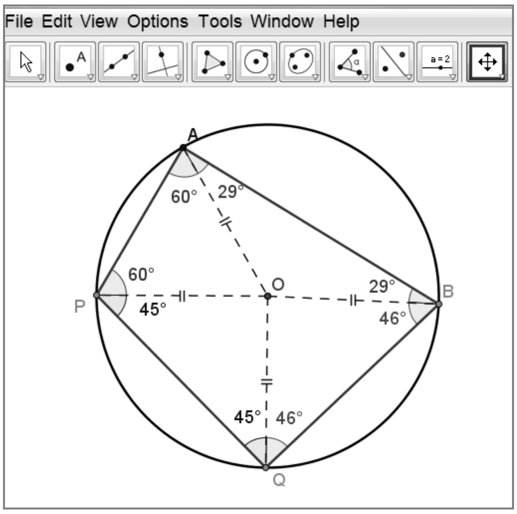


Figure 1

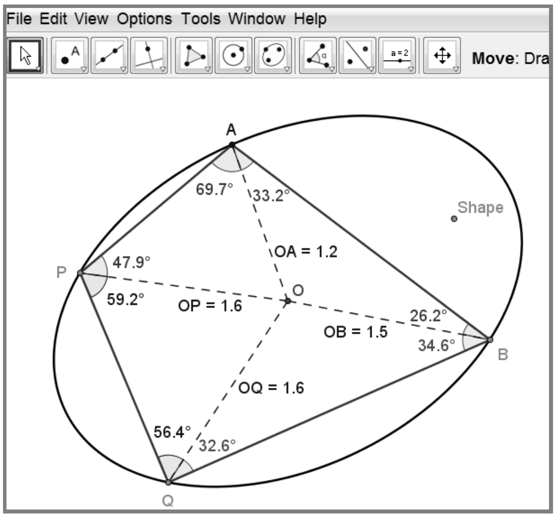


Figure 2

Figure 1 is a typical dynamic sketch that shows the sum of its opposite angles is always equal to 180°. By dragging the vertices A, B, Q or P, students may discover one of the circle theorem: sum of the opposite angles in a cyclic quadrilateral is always 180° (e.g. refer to Figure 1, $\angle APQ + \angle ABQ = 60° + 45° + 29° + 46° = 180°$). However it does not implies that students can deduce a proof of the theorem. Although the radii OA, OB, OQ and OP are drawn in the dynamic sketch, some of the students can still fail to alert their importance in order to formulate the proof. If this happens, I will usually provide another dynamic sketch which is not typical (refer to Figure 2). In this sketch, students can drag the point "Shape" to change the circle into ellipse. They soon will discover that the sum of its opposite angles of ABQP is not always 180° anymore. Then I will ask students to explain the following question: "Why the sum of the opposite angles of the quadrilateral ABQP is always 180° in Figure 1 but not in Figure 2? What is the *major difference between two figures*?" Usually the discussion between students and the teacher will as follows:

Student: Because Figure 1 is a circle but Figure 2 is not.

Teacher: Yes, the shapes are different. By the way what are the differences about the lengths and angles in the figures?

Student: There are four isosceles triangles in Figure 1 but not in Figure 2.

Teacher: Why the four triangles in Figure 1 must be isosceles?

Student: OA, OB, OQ and OP are radii of the circle, they must be equal in length.

Teacher: Are they important for us to show that the sum of the opposite angles in a cyclic quadrilateral is always 180°?

Student: Yes, because without them, the sum of opposite angles will not be 180° in Figure 2.

The above discussion can be seen as a preliminary explanation of the theorem. By dragging the vertices in Figure 1 and focusing on the contrast between Figure 1 and Figure 2, students usually can discover why the property OA = OB = OQ = OP is essential to explain the theorem. Once the students notice this property, they may be able to deduce the proof which is similar as follows:

$$\begin{aligned}
&OA = OB && \Rightarrow && \angle OAB = \angle OBA = a \\
&OB = OQ && \Rightarrow && \angle OBQ = \angle OQB = b \\
&OQ = OP && \Rightarrow && \angle OQP = \angle OPQ = c \\
&OP = OA && \Rightarrow && \angle OPA = \angle OAP = d
\end{aligned}$$

$$\text{Since} \quad 2a + 2b + 2c + 2d = 360^\circ$$

$$\text{Therefore} \quad a + b + c + d = 180^\circ$$

$$\text{i.e.} \quad \angle APQ + \angle ABQ = 180^\circ \quad \text{and} \quad \angle PAB + \angle PQB = 180^\circ$$

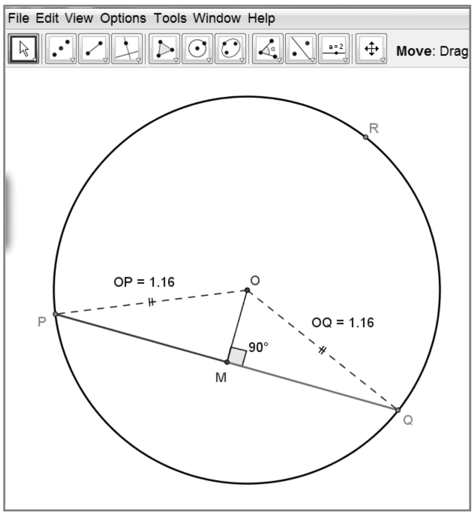
In Figure 1, the property OA = OB = OQ = OP was kept unchanged (invariant) throughout dragging, and it can be defined as invariant properties in the dynamic sketch. Such kind of invariant properties are usually essential for deductive justification of geometric theorems, but they may not easily be noticed (discerned) by students. By introducing the sketch in Figure 2, students can change the circle to ellipse (and vice versa) and hence notice the relations "OA = OB = OQ = OP $\Rightarrow$ $\angle APQ + \angle ABQ = 180°$" and also "if OA, OB, OQ and OP are not all equal, then $\angle APQ + \angle ABQ \neq 180°$". Such a contrast facilitates students to discern the critical properties. Leung (2008) states that one of the DGE's power is to equip with the ability to retain a background geometrical configuration, meanwhile particular parts in the whole configuration can be selectively brought to the fore via dragging. When parts (such as angles and lengths) are being focused and temporarily demarcated from the whole (background) in DGE, a discernment of invariants may be come out. Based on the invariants discerned, students may initiate the deductive justification of theorems. In order to show how the discernment of invariants crucial to facilitate students' deductive thinking in DGE, a study was conducted in a form four geometry lesson.

Case study

35 form four students with weaker ability in mathematics participated in this study. Three pre-constructed dynamic geometry sketches were prepared for students to investigate the theorem "line from center perpendicular to chord bisect chord". The three sketches differed by the invariant properties (those properties will not be changed under dragging) in the design. The details of each sketch were as follows:

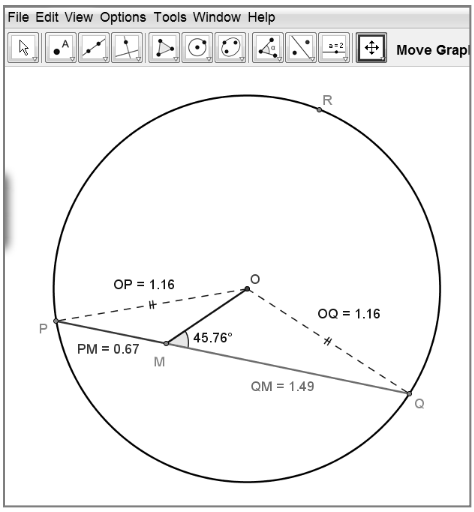
(The positions of P and Q can be dragged to move along the curve PQR freely in all three sketches.)

(a) Sketch 1



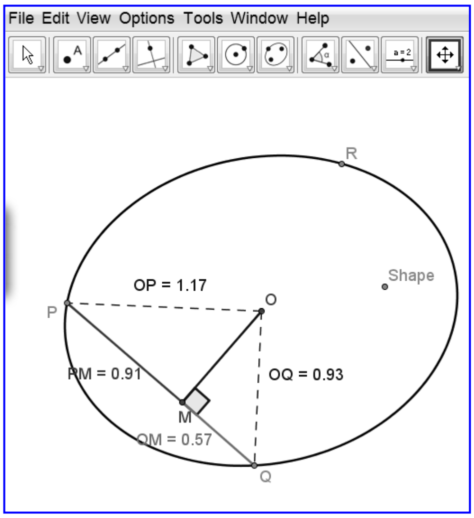
Invariants:	1. OP = OQ 2. ∠OMP = 90° 3. PM = QM
Variants:	1. The size of the circle (vary by dragging R)

(b) Sketch 2



Invariants:	1. OP = OQ
Variants:	1. Lengths of PM and QM 2. Value of ∠OMP (vary by dragging the point M)

(c) Sketch 3



Invariants:	1. ∠OMP = 90°
Variants:	1. Lengths of OP and OQ 2. Lengths of PM and QM (vary by changing the shape of PQR through dragging the point “Shape”)

All of the 35 students were divided into two groups. In Group 1, students were assigned to explore the theorem by using sketches 1, 2 and 3 in sequence; while in Group 2, the order of the sequence was reversed, i.e. students were assigned to explore the theorem by using the sketches 3, 2 and 1 in sequence. The details are as follows:

	1st Round (1st 10 minutes)	2nd Round (2nd 10 minutes)	3rd Round (last 10 minutes)
Group 1	Sketch 1	Sketch 2	Sketch 3
Group 2	Sketch 3	Sketch 2	Sketch 1

A worksheet was given to students in each round (see appendix I). Each worksheet included directions for manipulating the sketch, a statement of the problem, a probing question (as delivering some hints or guidelines to students) and finally an open-ended question which asked the students to express their findings in words (students can choose to answer either in English or Chinese). The time allowed for each round is 10 minutes. The students were requested to work individually and discussion was not allowed. When the time was up, all students preceded to the next round. All the worksheets were collected by the teacher after each round.

Results and discussion

At the beginning of each round, most of the students had spent around 3 to 5 minutes to drag on the points in the sketch for investigation. Then they commonly spent rest of the time to explain why PM = QM (i.e. the chord PQ is bisected). The flow of the work of students can be defined as an "experimentation-conjecturing-explanation" cycle (Or, 2005). Almost all students in both groups had no difficulties in the experimentation and conjecturing stages. For example, all students in Group 1 could highlight the visual information in the experimentation stage by using Sketch 1, such as:

- The figure is symmetrical
- There is an isosceles triangle
- PQ is bisected / M is the mid-point
- OM is perpendicular bisector

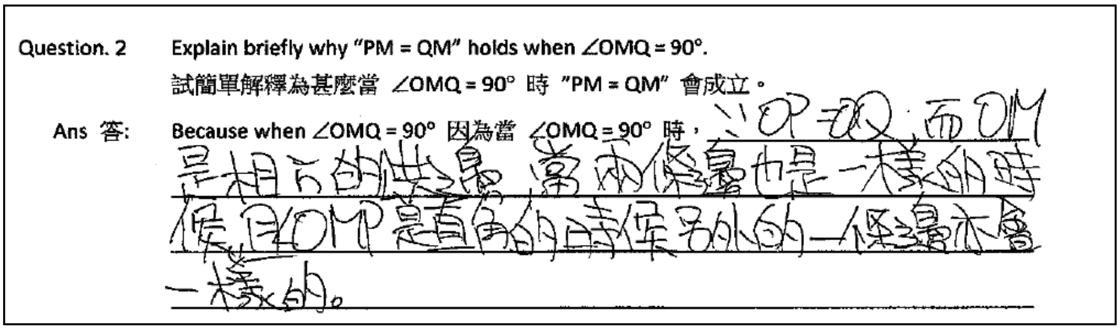
All of the above were general properties of Sketch 1, but they were too general for students to deduce the proof of the theorem "line from center perpendicular to chord bisect chord". Some students were misunderstanding the term "explain" in mathematics and they were not actually explaining why $\mathrm{PM} = \mathrm{QM}$ deductively. Instead, they thought that "M is the mid-point", "OM is perpendicular bisector", etc. were good enough to explain why PM = QM. On the other hand, lots of students had noticed the properties that were more important to deduce PM = QM. Those properties were classified as critical properties and listed as follows:

- Radius are same (i.e. OP = OQ)
- Right angles are presented (i.e. ∠PMO = 90° or ∠QMO = 90°)
- There is a common side (i.e. OM = OM)

The above properties found in the three sketches can be summarized as follows:

Catalogue	Description	Kept invariant (unchanged during dragging) in		
		Sketch 1	Sketch 2	Sketch 3
General properties (which were less useful for students to deduce proof)	The figure is symmetrical	✓		
	There is an isosceles triangle	✓		
	PQ is bisected / M is the mid-point	✓		
	OM is perpendicular bisector	✓		
Critical properties (which were more useful for students to deduce proof)	Radius are same (OP = OQ)	✓	✓	
	Right angles are presented (∠PMO = ∠QMO = 90°)	✓		✓
	There is a common side (OM = OM)	✓	✓	✓
	∴ ΔOMQ ≅ ΔOMP	✓		

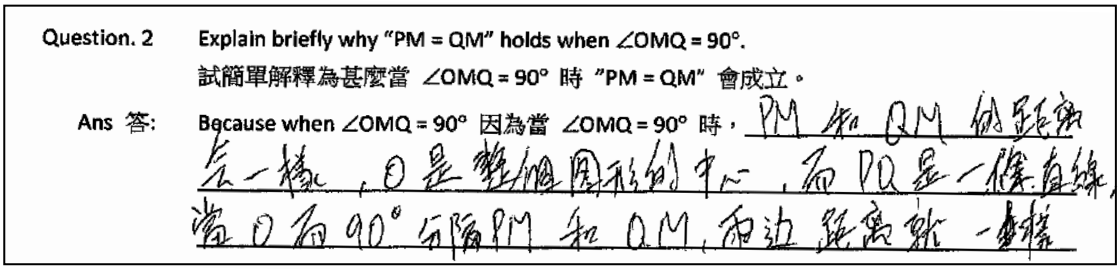
Two-thirds of the 35 students had noticed some critical properties above in the sketches. 10 students pointed out that there was a pair of congruent triangles and it could be used to prove PM = QM, while 2 students had worked out an incorrect “SSA” proof, and other 2 students had worked out the correct “RHS” proof actually. For example, David had identified all the critical properties $OP = OQ$, $OM = OM$ and ∠OMP = 90°, but he failed to link them logically to explain why PM = QM:



Question. 2 Explain briefly why "PM = QM" holds when ∠OMQ = 90°.
試簡單解釋為甚麼當 ∠OMQ = 90° 時 "PM = QM" 會成立。
Ans 答: Because when ∠OMQ = 90° 因為當 ∠OMQ = 90° 時，∵OP=OQ，而OM是相同的一條邊，當兩條邊也是一樣的時候，因∠OMP是直角的時候，另外的一條邊不會一樣的。

Work of David in Group 1, using Sketch 2 in the 2nd round

In Group 2, some students thought that their observation on single critical property was good enough to explain the theorem. For example, Susan tried to explain why PM = QM by just mentioning her inductive exploration on $\angle OMQ = 90^\circ$:



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試簡單解釋為甚麼當 ∠OMQ = 90° 時 "PM = QM" 會成立。
Ans 答: Because when ∠OMQ = 90° 因為當 ∠OMQ = 90° 時，PM和QM的距離是一樣，O是整個圖形的中心，而PQ是一條直線，當O有90°分隔PM和QM，兩边距離就一樣

Work of Susan in Group 2, using Sketch 2 in the 2nd round

After the discussion with the student, Susan clarified her explanation as follows:

> *"When I drag on M in Sketch 2, the position of M is moving along PQ. When ∠OMQ is 90°, the distance of PM and QM are same. Therefore I know that when ∠OMQ is right angle, PQ is bisected and hence* $PM = QM$*."*

Susan was one of the students who intended to justify PM = QM inductively but not deductively. Another student, Betty, also tried to explain why PM = QM inductively. However compared with Susan, the work of Betty seemed to be more logical:

Question. 2 Explain briefly why "PM = QM" holds when $\angle OMQ = 90°$.

試簡單解釋為甚麼當 $\angle OMQ = 90°$ 時 "PM = QM" 會成立。

Ans 答: Because when $\angle OMQ = 90°$ 因為當 $\angle OMQ = 90°$ 時，OQ 和 OP 的斜率一樣，再用畢氏定理就可知道 PM = QM，而且 OP = OQ OPQ 是個等腰三角形，當把 O 拉去 M，把三角形平均分成一半，底線就會一樣

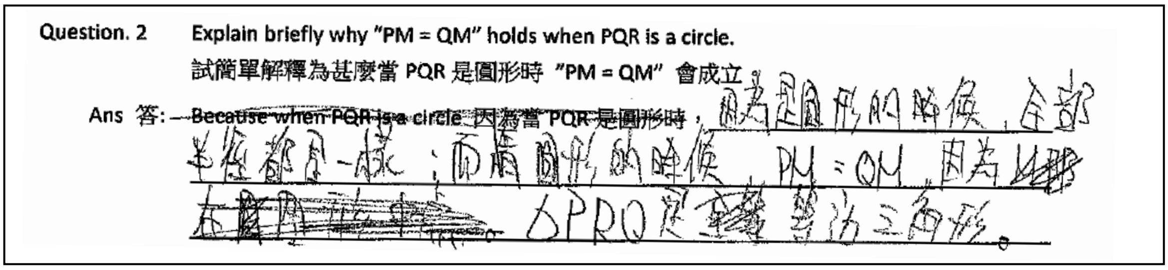
Work of Betty in Group 2, using Sketch 2 in the 2nd round

Once Betty noticed OP = OQ (although failed to mention the reason) and dragged the point M to the middle of PQ, she tried to show PM = QM by applying her prior knowledge learnt in form three – a line which can bisect an isosceles triangle into two equal halves must be the perpendicular bisector of a side of the triangle. Although the work of Betty was presented poorly, it can still be considered as "semi-deductive" justification, which is in-between inductive exploration and deductive justification.

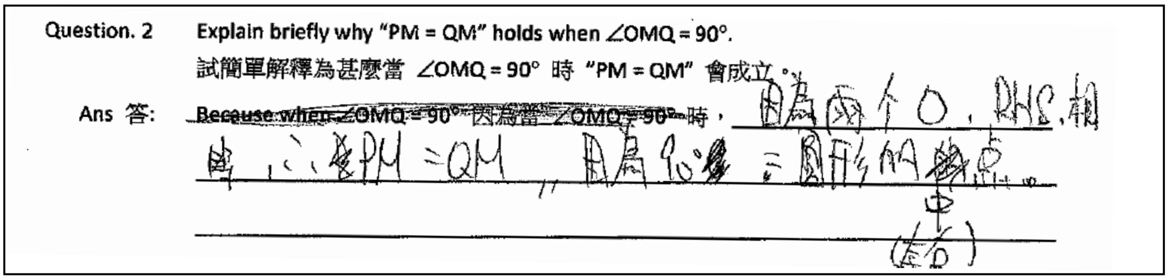
John was one of the students who also failed to show the correct proof, but his work was not worthless. Instead, John had demonstrated the transition from inductive exploration to deductive justification throughout the lesson. In the first round, John could only notice that OP = OQ was essential to show $\mathrm{PM} = \mathrm{QM}$, and delivered a justification which was fully inductive (and also incorrectly). When he proceeded to Sketch 2, he could notice $\angle OMQ = 90°$ (right angle) was also important and he began to explain the theorem by using the term "RHS". Finally at the third round, he tried to work out the proof of the theorem:

$$\angle QMO = 90°$$
$$\Delta OMQ \cong \Delta OMP \text{ (R.H.S.)}$$
$$\therefore \quad PM = QM$$

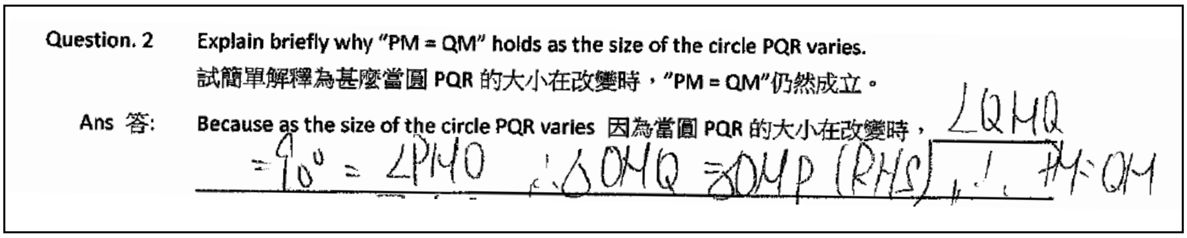
Although his proof was incomplete eventually, he had showed a transition from inductive exploration to deductive justification in DGE. It is a good sign to show that students are benefited from DGE and acquire a logical understanding of theorems.


Question. 2 Explain briefly why "PM = QM" holds when PQR is a circle.
試簡單解釋為甚麼當 PQR 是圓形時 "PM = QM" 會成立。
Ans 答: ~~Because when PQR is a circle 因為當 PQR 是圓形時，~~ 因為是圓形的時候，全部半徑都是一樣；而當圓形的時候，PM = QM，因為 △PRQ 是等邊三角形。

Work of John in Group 2, using Sketch 3 in the 1st round

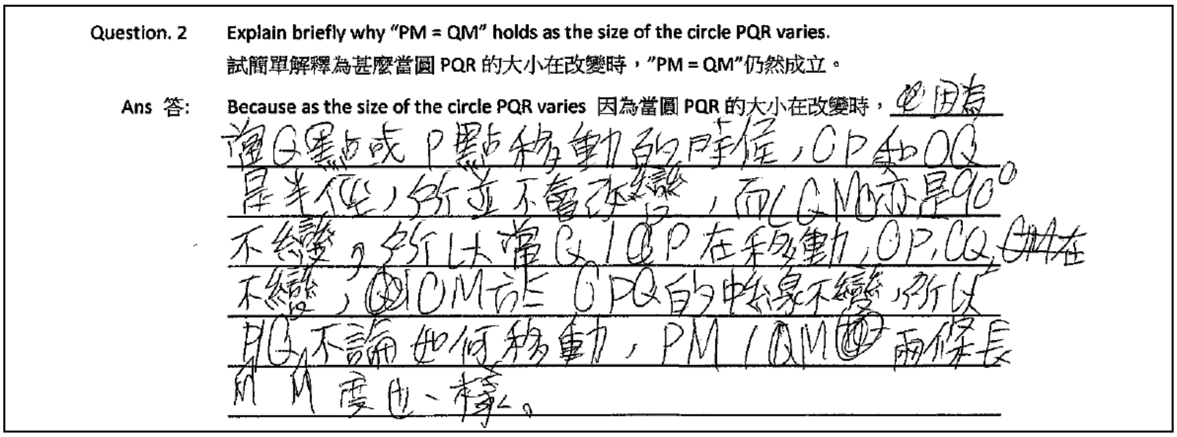

Question. 2 Explain briefly why "PM = QM" holds when ∠OMQ = 90°.
試簡單解釋為甚麼當 ∠OMQ = 90° 時 "PM = QM" 會成立。
Ans 答: ~~Because when ∠OMQ = 90° 因為當 ∠OMQ = 90° 時，~~ 因為兩个△，RHS，相同，∴ PM = QM，因為 O = 圓形的中點(中心)。

Work of John in Group 2, using Sketch 2 in the 2nd round

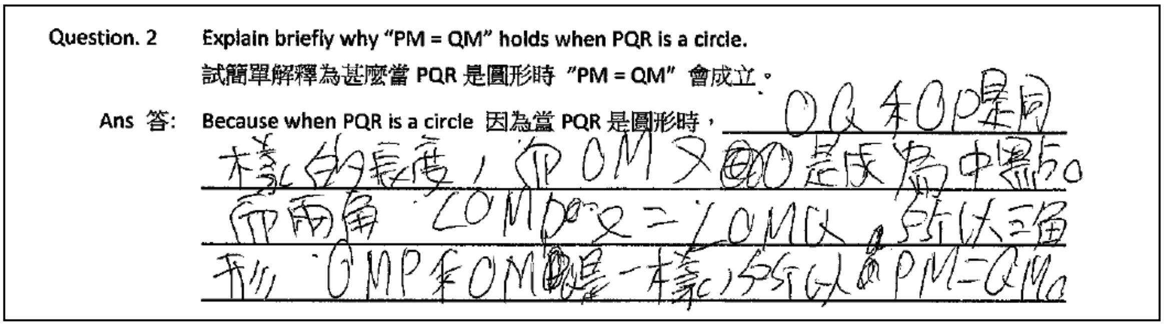

Question. 2 Explain briefly why "PM = QM" holds as the size of the circle PQR varies.
試簡單解釋為甚麼當圓 PQR 的大小在改變時，"PM = QM"仍然成立。
Ans 答: Because as the size of the circle PQR varies 因為當圓 PQR 的大小在改變時，∠QMO = 90° = ∠PMO ∴ △OMQ ≅ △OMP (RHS) ∴ PM = QM

Work of John in Group 2, using Sketch 1 in the 3rd round

Similarly, Sam in Group 1 did not notice the congruent triangles in the first two rounds until the last round, and his work was transited from inductive to deductive eventually:


Question. 2 Explain briefly why "PM = QM" holds as the size of the circle PQR varies.
試簡單解釋為甚麼當圓 PQR 的大小在改變時，"PM = QM"仍然成立。
Ans 答: Because as the size of the circle PQR varies 因為當圓 PQR 的大小在改變時，因為當Q點或P點移動的時候，OP和OQ是半徑，所以並不會改變，而∠QMO亦是90°不變，所以當Q、P在移動，OP, OQ 在不變，而OM是OPQ的中線不變，所以PQ不論如何移動，PM、QM兩條長的也是一樣。

Work of Sam in Group 1, using Sketch 1 in the 1st round


Question. 2 Explain briefly why "PM = QM" holds when PQR is a circle.
試簡單解釋為甚麼當 PQR 是圓形時 "PM = QM" 會成立。
Ans 答: Because when PQR is a circle 因為當 PQR 是圓形時，OQ和OP是同樣的長度，即OM又是成為中點。即兩角∠OMP又=∠OMQ，所以三角形OMP和OMQ是一樣，所以PM=QM。

Work of Sam in Group 1, using Sketch 3 in the 3rd round

In the first round, Sam dragged the points in the sketch and noticed that both properties OP = OQ and ∠QMO = 90° were invariants while dragging. Although he had mentioned OM is median of ΔOPQ, he failed to use it to explain PM = QM logically. However in the third round, he stated that ΔOMP≅ΔQMQ, which could be used to deduce PM = QM. Although he had not written down any formal proof, his work had shown the transition from inductive to deductive clearly.

What is the key in facilitating the justification of John and Sam from inductive to deductive during the lesson? Both of them failed to mention the congruent triangles in the first round. How could they alert the existence of congruent triangles and use them to deduce PM = QM? It is believed that the different designs of invariants in the sketches provided different perspectives for students to notice the critical properties (which are OP = OQ, ∠QMO = 90° and hence ΔOMP ≅ OMQ). By examining the invariant configuration in the three sketches and the critical properties noticed by students during the lesson, the results can be summarized as the following tables:

Summary of the results in Group 1

Critical properties	Properties kept invariant during dragging			No. of students noticed the properties (and wrote them on the worksheet)		
	1st Round (Sketch 1)	2nd Round (Sketch 2)	3rd Round (Sketch 3)	1st Round (Sketch 1)	2nd Round (Sketch 2)	3rd Round (Sketch 3)
OP = OQ	✓	✓		10	12	15
∠PMO = 90° or ∠QMO = 90°	✓		✓	5	9	9
OM = OM	✓	✓	✓	4	6	7
ΔOMP ≅ OMQ	✓			2	2	4

Summary of the results in Group 2

Critical properties	Properties kept invariant during dragging			No. of students noticed the properties (and wrote them on the worksheet)		
	1st Round (Sketch 3)	2nd Round (Sketch 2)	3rd Round (Sketch 1)	1st Round (Sketch 3)	2nd Round (Sketch 2)	3rd Round (Sketch 1)
OP = OQ		✓	✓	10	15	15
∠PMO = 90° or ∠QMO = 90°	✓		✓	1	10	12
OM = OM	✓	✓	✓	1	3	4
ΔOMP ≅ OMQ			✓	2	5	6

It is believed that different invariant configuration in dynamic sketches facilitated working of students from induction to deduction when they noticed more critical properties throughout the lesson. As a result, the learning outcome of the students by using all the three sketches is better than using one (Sketch 1 or Sketch 3) solely in the first round. It can be explained by using the theory of variation.

If you want to let the students realize "$OP = OQ$ and $\angle QMO = 90° \Rightarrow PM = QM$" in dynamic geometry sketch, you can keep the properties $OP = OQ$ and $\angle QMO = 90°$ invariant (i.e. Sketch 1). On the other hand, you can use variation, for example:

"$OP = OQ$ but $\angle QMO \neq 90° \Rightarrow PM \neq QM$" (i.e. Sketch 2) or
"$\angle QMO = 90°$ but $OP \neq OQ \Rightarrow PM \neq QM$" (i.e. Sketch 3)

The different invariant configuration in the sketches can provide a contrast for the students to notice the role of critical properties in the deduction of theorems. Once the students notice such an invariant relation "$OP = OQ$ and $\angle QMO = 90° \Rightarrow PM = QM$", (i.e. $PM = QM$ holds only when $OP = OQ$ and $\angle QMO = 90°$) during dragging, they have discerned the invariant which can help themselves to proceed from inductive exploration to deductive justification. In order to facilitate such a discernment of invariant, variation is required. Variation is about what changes, what stays constant and what the underlying rule is. Leung (2012) described mathematical experience as "the discernment of invariant pattern concerning numbers and/or shapes and the re-production or representation of that pattern." In particular, discernment of critical features occurs under systematic interaction between learners and the thing to be learnt, and variation is an agent that generates such interaction. Leung (2011) suggested that the drag-mode in DGE seems to open up a new methodology and even a new discourse to acquire geometrical knowledge alternative to the traditional Euclidean deductive reasoning paradigm. It is suggested that teachers should try to provide more variation (e.g. contrast on invariants) in designing dynamic sketches in order to initiate students' thinking and hence facilitate the bridging of inductive nature of DGE and deductive justification in Euclidean geometry.

Conclusion

Although direct teaching of the proof on theorems (Euclidean geometry) is much easier and efficient in geometry lessons, it can only facilitate students' procedural understanding but not conceptual understanding. Dynamic geometry sketches is powerful for the teachers to facilitate students' conceptual understanding of geometrical knowledge through the transition from inductive exploration to deductive justification in DGE. With the different design of the invariant configuration in the dynamic sketches, teachers are possible to enhance the learning effectiveness and outcome of the students in geometry lessons. Last but not least, variation in DGE may facilitate the higher order thinking of the students:

Question. 2 Explain briefly why "PM = QM" holds when PQR is a circle.
試簡單解釋為甚麼當 PQR 是圓形時 "PM = QM" 會成立。

Ans 答: Because when PQR is a circle 因為當 PQR 是圓形時，中心點是O，半徑會是一樣，例如 OP=[illegible]=OQ，由於有相同的長度及角度，PM 便會等於 MQ，但當是橢圓形時，並不是每個半徑都一樣，因此 PM ≠ MQ。

Work of Sophia in Group 1, using Sketch 3 in the 3rd round

Sophia had already delivered the correct proof in the first and second round. Once she saw the ellipse in the third round, she reflected about why $\mathrm{PM} = \mathrm{QM}$ was true only when PQR was a circle. She understood why there are so many theorems about circle but not ellipse as radii plays an important role to deduce those theorems. It is glad to see that hands-on experiences in DGE facilitated a reflective thinking of students.

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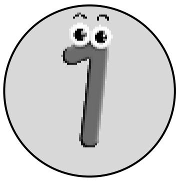
Appendix I

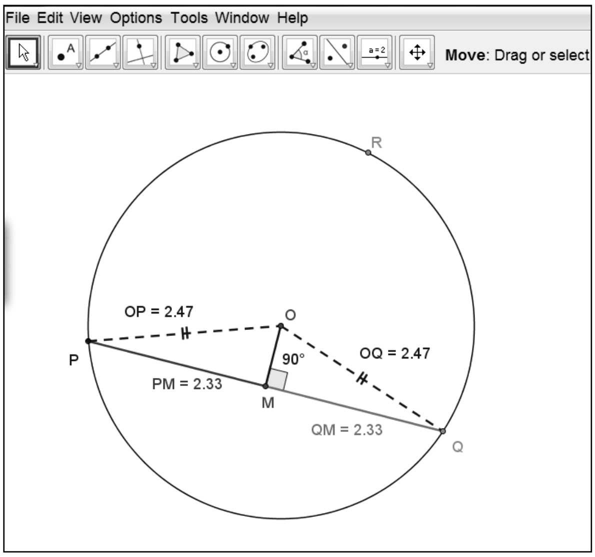
Worksheet used in Sketch 1



Geogebra In-class Exploration Activity (I)

Name 姓名: _______________ Class 班別 : ___________ Date 日期 : __________

Locate the file "Z:\Geogebra\task1.ggb" and double click to open it.
雙擊並打開檔案 "Z:\Geogebra\task1.ggb"。



Given **fixed** conditions in the sketch
在圖中已知的**固定**條件 :

1. PQR is a circle with center O.
   PQR 是一個圓形，其圓心為 O。
2. $\angle OMQ = 90°$.
3. PM = QM.

Your task 你的任務 :

Drag the point P to change the size of the circle as you like. (You can drag the points P and Q around the circle also.)
拖曳 P 點去改變圓形的大小。(你亦可以拖曳 P 及 Q 點沿著圓形上走動。)

Question. 1 Does "PM = QM" hold as the size of the circle PQR varies? Yes / No
當圓 PQR 的大小在改變時，"PM = QM" 是否仍然成立? 是 / 否

Question. 2 Explain briefly why "PM = QM" holds as the size of the circle PQR varies.
試簡單解釋為甚麼當圓 PQR 的大小在改變時，"PM = QM"仍然成立。

Ans 答: Because as the size of the circle PQR varies 因為當圓 PQR 的大小在改變時，

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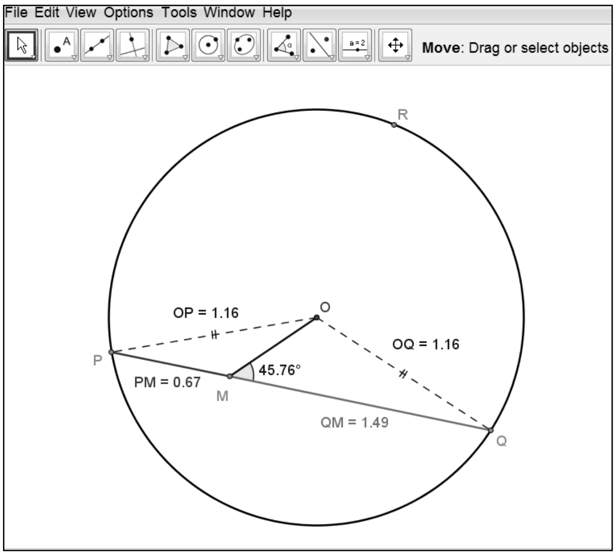
Worksheet used in Sketch 2

Geogebra In-class Exploration Activity (II)

Name 姓名: ______________ Class 班別 : __________ Date 日期 : _________

Locate the file "Z:\Geogebra\task2.ggb" and double click to open it.
雙擊並打開檔案 "Z:\Geogebra\task2.ggb"。



Given **fixed** conditions in the sketch
在圖中已知的**固定**條件 :

1. PQR is a circle with center O.
   PQR 是一個圓形，其圓心為 O。

Your task 你的任務 :

Drag the point M to change value of ∠OMQ as you like. (You can drag the points P and Q around the circle also.)
拖曳 M 點去改變∠OMQ 的大小。(你亦可以拖曳 P 及 Q 點沿著圓形上走動。)

Question. 1 What is the value of ∠OMQ when "PM = QM" holds?
當 "PM = QM" 成立時，∠OMQ 的值是多少? ∠OMQ = __________

Question. 2 Explain briefly why "PM = QM" holds when ∠OMQ = 90°.
試簡單解釋為甚麼當 ∠OMQ = 90° 時 "PM = QM" 會成立。

Ans 答: Because when ∠OMQ = 90° 因為當 ∠OMQ = 90° 時，

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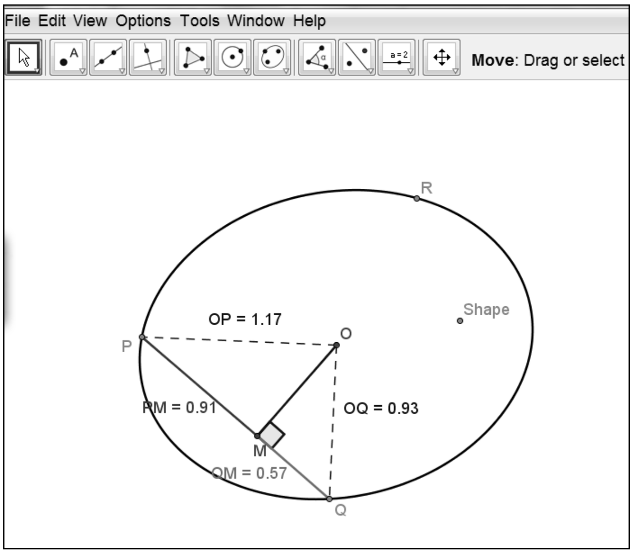
Worksheet used in Sketch 3

Geogebra In-class Exploration Activity (III)

Name 姓名: ______________   Class 班別 : __________   Date 日期 : _________

Locate the file “Z:\Geogebra\task3.ggb” and double click to open it.
雙擊並打開檔案 “Z:\Geogebra\task3.ggb”。



Given **fixed** conditions in the sketch
在圖中已知的**固定**條件 :

1. $\angle OMQ = 90°$.

Your task 你的任務 :

Drag the point “Shape” to change the shape of PQR. (You can drag the points P and Q around the circle also.)
拖曳“Shape” 點去改變 PQR 的形狀。(你亦可以拖曳 P 及 Q 點沿著圓形上走動。)

Question. 1 What is the shape of PQR when “PM = QM” holds?
當 ”PM = QM” 成立時，PQR 的形狀是甚麼? PQR is 是 __________

Question. 2 Explain briefly why “PM = QM” holds when PQR is a circle.
試簡單解釋為甚麼當 PQR 是圓形時 ”PM = QM” 會成立。

Ans 答: Because when PQR is a circle 因為當 PQR 是圓形時，

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