Tool-based Pedagogy for School Mathematics: A Research Proposal

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Abstract

This paper outlines the rationale and theoretical background of a research agenda that aims to develop a teaching and learning approach for school mathematics using concrete tools. The purpose is to arouse mathematics teachers' interest to participate in this research.

School mathematics has often been regarded as a subject that is rigid and abstract; nevertheless, it is commonly agreed as an essential component in school curriculum to foster students' ability to reason. Teachers and students often find difficult to make teaching and learning of mathematics interesting in ways to relate it to concrete daily experiences. This may due to the tradition that mathematics should be taught in a deductive and authoritative approach. In the Hong Kong New Senior Secondary Curriculum, mathematics is one of the four mandatory subjects taken by students, which means that all students have to study mathematics throughout their school lives. This reform set off a need to refresh the teaching and learning of mathematics to a more fit-for-all state of affair where all students could enjoy the intellectual beauty that mathematics provides.

In a traditional Hong Kong school mathematics classroom, the teacher usually presents known abstract mathematical facts to students that are often symbolic in nature, and students use these facts to solve problems. This approach no doubt is effective to enable students to prepare for examination and could be conducive to develop students' power of (deductive) reasoning, but it does not make explicit an epistemological perspective that mathematics can be experienced and learnt through the use of real objects and that abstract ideas can

be acquired through a process of personal or collaborative inquiry that are relevant to students' daily experience. Research has shown that mathematical experiences can be mediated through tools. Tools are commonly considered as resources susceptible of enhancing the teaching-learning activity. Tools here are interpreted as physical or virtual artefacts that have the potential to mediate between mathematical experience and mathematical understanding. For examples, wooden unit blocks or abacus-like tool helps children develop arithmetic, and interactive dynamic geometry platforms enhance students' understanding of geometry. Through a designed integration of appropriate tools into the teaching and learning of mathematics, students are opened to an inquiry learning environments where they can experience and make sense of abstract mathematical concepts and consequently construct them into personal or collaborative knowledge. This kind of constructivism using tools has been shown to greatly empower students' mathematical ability. In this connection, teachers play a complex and critical role to exploit the pedagogical potentials of the tools.

As stated in the *The New Senior Secondary Mathematics Curriculum and Assessment Guide (Secondary 4 - 6)* (Hong Kong Education Bureau, 2007), one of the rationales for students to study mathematics is that "mathematics is a powerful means in a technology-oriented and information-rich society to help students acquire the ability to communicate, explore, conjecture, reason logically and solve problems using a variety of methods." Furthermore, a mathematics curriculum aim is to develop in students "the ability to think critically and creatively, to conceptualise, inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life" (Hong Kong Education Bureau, 2007). To achieve these attributes in our present day classrooms, the teaching and learning of school mathematics should seek to utilize different resources to empower students to develop the communication, exploration, conjecture-making and reasoning skills. The traditional pedagogy for Hong Kong school mathematics that emphasizes mainly on abstract symbolic deductive teaching and learning, though effective in preparing able students to excel, is often regarded by students as uninteresting. Local research

on students' attitude towards mathematics had shown that "many students described the mathematics class as boring. Students repeatedly asked for the introduction of more lively teaching methods…They suggested that more…teaching aids, real life applications … would be helpful." (Wong, et al., 2002) and they unanimously agreed to the statement "When learning a new topic, I wish that I could think it through by myself first and not having the teacher telling me everything" (Wong et al., 2001). This student attitude towards mathematics teaching and learning is often confirmed in international comparative assessments like TIMSS (Trends in International Mathematics and Science Study). Hong Kong students' performance in TIMSS 2007 was in the top tier among 58 participating countries. However, in the Hong Kong Press Conference for TIMSS 2007, it was stated that

> "It is interesting to note that although Hong Kong … students do very well in their mathematics … achievement, they are less likely to enjoy and to have high values and high self-confidence in learning mathematics … when compared to students in other parts of the world … Hong Kong students … are generally lower than the international averages on their high values, high positive affect and high self-confidence in learning mathematics ..."
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> TIMSS 2007 Hong Kong Press Release
> http://web.edu.hku.hk/outreach/media/docs/media/081210_TIMSS_R_E.pdf

Thus students desire and need a more pragmatic, enjoyable way to learn mathematics. This sentiment would not change until innovative pedagogy appears to address the issue. In this connection, a pedagogy supported by the use of concrete tools (physical or virtual) to mediate mathematical knowledge is a potential candidate to this end, motivating students to engage in a deep level of mathematics learning via hand-on inquiry exploration. To develop a tool-based approach to teach and learn mathematics is the central focus of this project.

A structural use of concrete objects (manipulatives) in mathematics teaching and learning was developed by Zoltan Dienes (1960, 1971). For example, Dienes designed the Multibase Arithmetic Blocks (known world-wide

as Dienes' Multibase Blocks) to provide children with multiple embodiments of the structure of our place-value number system. John Olive in a reflection of his personal odyssey in the use of artifacts, materials and tools for learning and teaching mathematics demonstrated "how the design and use of structured materials for mathematics learning, pioneered by Dienes in the 1960's and 70's, have been both assimilated and transformed by the new technologies developed over the past 40 years." (Olive, 2008) Uttal et al (1997) offered a perspective on the use of concrete object to teach mathematics where manipulatives are considered as symbols and teachers intend for them to stand for or represent a concept or written symbol. A US research study explored problem solving in elementary classrooms focusing on how children use (perform tasks) manipulatives (or tools) in problem solving while working on mathematical tasks concluded that "if manipulative use becomes an integral part of the academic structure for all students in mathematics classrooms, it may keep more students in higher-level math classes through college and beyond." (Kelly, 2006) The use of tools in mathematics classroom has been a common practice for teachers; however, there is yet a pragmatic theory on how to transform a tool into a pedagogical instrument in the mathematics classroom.

Bartolini-Bussi and Mariotti (2008) have been developing, and experimenting, a pedagogical theory of using artefacts in mathematics classrooms. An artefact in a mathematics classroom can be any tool that has the potential to mediate a piece of mathematical knowledge. A familiar example would be a pair of compasses which embodied the concept of circle and distance. Students can experiment with it to understand the mathematics of circle with properly designed teaching tasks. Nevertheless, the relationship between artifacts and knowledge is complex. Rabardel's (1995) theory of *instrumental genesis* was an explication of how the usage of a tool can be turned into a cognitive instrumentation process for knowledge acquisition. This is a theory that focuses on how a learner develops a scheme of usage for a particular tool while s/he is using it to solve a problem. Pedagogically, this scheme can be attached to the tool to make it into an instrument for teaching and learning. In another vein, a Vygotskian approach would see artefacts as psychological tools

in the context of social and cultural interaction developed through the *zone of proximal development* and *internalization* processes (Vygotsky; 1978, 1981). A concrete manipulative can be transformed into a mind-tool through a carefully designed pedagogical environment. In particular, internalization is a *socio-semiotic* process. With these constructs as backdrop, Bartolini-Bussi and Mariotti develop a notion of *tool of semiotic mediation* for the mathematics classroom under which a tool takes on multiple pedagogical functions. One the one hand, personal meanings are related to the use of the tool while students accomplishing a task; on the other hand, mathematical meanings are related to the tool and its use. The dual relationships constitute *the semiotic potential of a tool*. An artefact is regarded as a tool of semiotic mediation if it is intentionally used by the teacher to mediate a mathematical content through designed didactical intervention. Thus teachers play the critical role in the process of tool mediation. In fact, the teacher is part of the mediation process. Figure 1 (Azrarello, Bartolini-Bussi, Leung, Mariotti & Stevenson, 2011) represents the classroom dynamic of the tool mediation process.

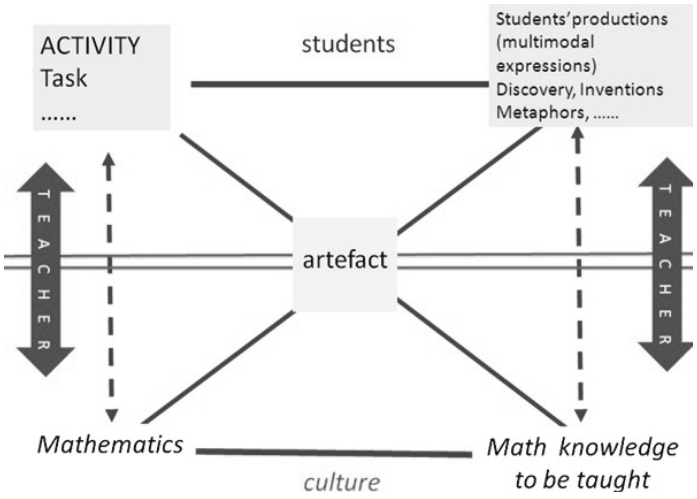

Figure 1

The dynamism consists of a network of interactions in which the teacher through the tool (artefact) acts as a mediator between the mathematics knowledge domain and the student activity task domain. Mariotti experimented with a group of Italian teachers implementing this framework using Cabri (a dynamic geometry software) as a tool (Mariotti and Maracci, 2010). In the teaching experiment, teachers implemented a sequence of classroom activities

using Cabri and orchestrated the classroom into a collective knowledge inquiry environment. The findings confirmed that the frame offered by the tool of semiotic mediation in a mathematics classroom serves as a means to support teacher's didactical action on the one hand, while on the other hand to promote students to a collective inquiry learning mode through accomplishing tasks. In particular, the researchers were able to develop a description of how the teacher can pilot a semiotic process based on the use of a tool and to outline the schemes that may be put in place to promote the evolution of mathematical knowledge during collective classroom discussion. In another research, Bartolini-Bussi and Maschieto discussed issues concerning primary and secondary mathematics teacher drawing on the activity of the Laboratory of Mathematical Machines at the Department of Mathematics of the University of Modena and Reggio Emilia (MMLab: www.mmlab.unimore.it). The discussion concluded that "concrete manipulation has to find a place both in mathematics classroom and in teacher education." (Bartolini-Bussi and Maschieto, 2008).

In the tool of semiotic mediation framework depicted in Figure 1, activity tasks play a critical role in the semiotic process. Leung (2011) has been developing a pedagogical framework for tool-based mathematics task design that has the potential to be merged into the tool of semiotic mediation framework. Leung's design framework consists of three nested epistemic modes of practice, discernment and discourse (Figure 2) which characterize three types of task design orientation that can support the tool of semiotic mediation framework. These nested modes can be seen as forming a semiotic process in acquiring mathematical knowledge using tool. The upward arrow in Figure 2 can be interpreted as a progressive transformation of an artefact from a concrete external object to a personal psychology tool during a well designed mathematics exploration activity.

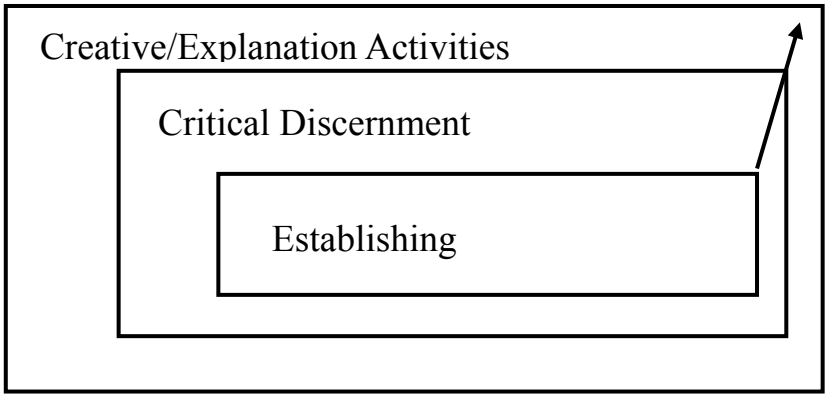

Figure 2

Combining these theoretical frameworks and realizing the critical role teachers play in the tool mediation process, a research is proposed that aims to investigate how Hong Kong teachers implement this tentative combined tool-based pedagogical model in their mathematics classrooms. A purpose of this research is to come up with a pragmatic teaching and learning approach where mathematics can be experienced and learnt through the use of real objects and that abstract ideas can be acquired through a process of personal or collaborative inquiry that are relevant to students' daily experience. A mathematical experience can be seen as "the discernment of invariant pattern concerning numbers and/or shapes and the re-production or re-presentation of that pattern." (Leung, 2010) Using tools to empower students to experience mathematics is a viable channel to expand the space of inquiry for students and to transform a mathematics classroom into a quasi-laboratory for experimentation. In order for students to access to this tool-based learning experience, teachers must be well-tuned to adapt to this novice didactical process. Hence, this research should start with developing together with teachers this tool-based pedagogy via training workshops and running school-based research lessons that utilize different tools. The collection of school-based research lessons will then form a rich source to build a pedagogical theory using the combined model as the foundation, and with this initiative, Hong Kong teachers will enter the world stage of mathematics education research.

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