對談講場

數學課外讀物書目

上期黃志華先生表達了一個愛看數學書者的願望。其實教育署輔導視學組數學科的同工也曾彙集了不少數學課外讀物的書目，包括

- Reference Books and Periodicals(1990)
- School Mathematics Club and Its Related Activities(1992)
- 小學數學參考書目(1992)
- 小學數學教學參考資料(1991)
- 透過小學數學課程推行公民教育(1991)

這些均可在北角百福道的數學教育資源中心找到。此外曾健華首席督學爲香港課外活動協會十週年文集《課外活動》（陳德恆編,廣角鏡出版社,1994 ）撰寫的"Organizing Mathematics Clubs in Secondary Schools"一文中亦附有一份中英文的書目。香港數理教育學會於 1986 年亦編有《數學學會導師資料冊》。筆者於《數學傳播》的「立體數學遊戲與空間想像力之訓練」（ 56 期,頁 78-96,1990 ）、「數學與課外活動」（ 62 期,頁 96-109,1992 ）、「遊戲與數學教育」（ 66 期,頁 52-68,1993 ）等文的參考書目中亦列舉了不少有關書籍，有興趣者可作參閱。

黃毅英

「名數除名數等於不名數」

上期「ΔABC≅ΔBCA?」一文見刊後，續有追問格式問題，特別涉及單位(名數)。小學時若遇「$20可買每個$4的蘋果若干?」須寫「20元/4元=5」，概「名數除名數等於不名數」；而對於「$20可買每米$4的布若干」則寫「20元/4元=5米」。不明所以之餘實須預先按題意定出「米」的單位。改4實爲4元/米，前則爲4元/個。其實解答在於達意，若格式不合規定首要追查背後思路是否紊亂，拘執表面恐於事無補。近閱Hall《代數學》一書，其解答不拘一格，卻旨在清晰表達思路。如159節例三「二人相隔27哩同時出發，若隨同一方向步行9時後相遇，若反方向則需3小時，求二人速度」一問，先設較快者每時行x哩，慢者每時y哩。若同方向每小時拉近x-y哩，反向則爲x+y哩。故有9(x-y)=27及3(x+y)=27。如是文字與算式交替，甚爲清楚。Durell《中學代數》6章例3解答中先舉

$$\frac{x-1}{3\frac{1}{2}}\text{時} + \frac{2}{3}\text{時} + \frac{x}{10\frac{1}{2}}\text{時} = 4\text{時},$$

再列

$$\frac{x-1}{3\frac{1}{2}} + \frac{2}{3} + \frac{x}{10\frac{1}{2}} = 4,$$

亦可參考。於實際教學運作而言，校內老師可先統一規定，明確事前告訴學生，以免惹起不必要的爭執和混亂。

黃毅英

More about "Directed Numbers"

In response to McClelland's (1994) and Wong's (1996) comments on "directed number" (see *EduMath*, *2*, p. 55), I like to supplement the following:

1. The use of the term "signed number" is apparently correct since there is indeed a sign before the number. Besides, it also points to the algebraic notion of completing a commutative ring in which we have, for every element, an additive inverse (called the 'negative' of that element).
2. The use of the term "directed number" is even more forward looking, though maybe less obvious, because the real numbers (or a bit more sophisticated, the rational numbers) so formed by adding 'negatives' constitute a one-dimensional vector space, an algebraic structure in which we can talk about directions! That comes the word 'directed.' Of course, in this one-dimensional vector space, there are two different directions only.

The study of the words invoked in mathematics quite often reveals meanings beyond surface interpretations and should not be taken light-heartedly. It helps learners to categorize concepts, build up links between them, fit them into a grand structure at a more macroscopic level, and hence open the way to further learning.

Fung Chun Ip
Hong Kong Institute of Education

A Visual Proof for $1^2 + 2^2 + \ldots + n^2 = \frac{1}{3}n(n+1)(n+\frac{1}{2})$ on the Backcover

Editor's Note: In selecting a diagram for the backcover of this issue, the editor came across a visual proof for the sum of squares formula which may not be familiar to many mathematics teachers. And it was later found that this proof was cited as due to our teacher and friend, Prof. Siu Man-Keung of the University of Hong Kong. When the editor wrote him to ask for permission to use this diagram , a history of it was revealed. Here it is.

Subject: Re: A visual proof by you for EduMath3 backcover
Date: Wed, 3 Dec 1996 15:27:52 GMT+8
Dear KM,

You suggest to use a picture in one of my papers for the backcover of EduMath and ask if I agree. Of course I would not mind. It feels so good to see one's work shown. In fact that picture is not entirely my own invention. It has a rather long story. The first time I drew it was when I published a paper with that as a diagram:

M.K. SIU, Pyramid, pile, and sum of squares, Historia Mathematica, 8 (1981), 61-66,

with the idea taken from a paper by XU Chun-fang (documented in my 1981 paper):

XU Chun-fang, From Chu Tong to Xi Ji and Si Yu Dou series (in Chinese), Shuxue Tongbao, 2 (1965), 45-49.

Xu surmised that YANG Hui did just that to discover the formula in his book "Chengchu Tongbian Suanbao" of 1274.

In a later volume of Historia Mathematica (vol.13 (1986)) the Russian historian of mathematics A.P. Youschkevitch added a comment on my paper, saying that the reconstruction appeared in a 1937 Russian translation (by S.Ya. Lourie) of the book "Vorlesungen ueber Geschichte der antiken mathematischen Wissenschaften, Vol I" by O. Neugebauer (1934), and noted that the explanation by Neugebauer is not as simple.

All these happened over a decade ago. Then, just last week I received a copy of the translated "A History of Chinese Mathematics" by J.-C. Martzloff which just came out of press (Springer-Verlag, 1997) with the original edition in French published in 1987. On p.303 I see that picture again! According to Martzloff, this explanation appeared in "Shuxue Yao" (Key to Mathematics) written by DU Zhigeng in 1681. I have not yet checked the original edition of the book, so I do not know whether the picture is really there in the 17th century book or not.

In 1984 I offered that picture as a response to the call of the editor for "proofs without words" for fill-in in the Mathematics Magazine. It was published in vol. 57 (1984), p.92. It was later collected into the book "Proofs Without Words" by R.B. Nelsen (Mathematical Association of America, 1993).

MK