

Converting Denary Numbers to Binary Numbers

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I suppose teachers would not have any difficulty in explaining the algorithm of converting binary numbers to denary numbers to their students. Say, to convert 1011001_2 , we can give a solution as follows:

$$\begin{aligned} & 1011001_2 \\ = & 2^6 + 2^4 + 2^3 + 1 \\ = & 64 + 16 + 8 + 1 \\ = & 89_{10} \end{aligned}$$

However, it is not easy for the other way round. In most textbooks, only the following algorithm, by using short division, was given when demonstrating how to convert 89_{10} to binary. No further explanation on this algorithm was given.

$$\begin{array}{r} 2 \) \ \underline{89} \\ 2 \) \ \underline{44} \ \text{.....}1 \\ 2 \) \ \underline{22} \ \text{.....}0 \\ 2 \) \ \underline{11} \ \text{.....}0 \\ 2 \) \ \underline{5} \ \text{.....}1 \\ 2 \) \ \underline{2} \ \text{.....}1 \\ \quad 1 \ \text{.....}0 \end{array}$$

$$\therefore 89_{10} = 1011001_2 .$$

Very often, students were not convinced even though I told them that this process was just a reverse process of the previous one with multiplication and addition being “reversed” by division and subtraction.

This year, I used a new method to solve the problem and eventually I found out that my method could also be used to explain the short-division algorithm.

My method is simple, to subtract the greatest power of 2 from the given (or remaining) value. The method is illustrated as follows.

$$\begin{aligned}
 & 89_{10} \\
 = & 64 + 25 && (2^6 = 64 < 89; \quad 2^7 = 128 > 89; \quad \text{and} \quad 89 - 64 = 25) \\
 = & 64 + 16 + 9 && (2^4 = 16 < 25; \quad 2^5 = 32 > 25; \quad \text{and} \quad 25 - 16 = 9) \\
 = & 64 + 16 + 8 + 1 && (2^3 = 8 < 9; \quad 2^4 = 16 > 9; \quad \text{and} \quad 9 - 8 = 1) \\
 = & 2^6 + 2^4 + 2^3 + 1 \\
 = & 1011001_2
 \end{aligned}$$

Then the short-division algorithm can be easily explained by comparing the following two computations.

$ \begin{array}{r} 2 \) \ \underline{89} \ \underline{\hspace{1cm}} \\ 2 \) \ \underline{44} \ \underline{\hspace{1cm}} \dots\dots\dots 1 \\ 2 \) \ \underline{22} \ \underline{\hspace{1cm}} \dots\dots\dots 0 \\ 2 \) \ \underline{11} \ \underline{\hspace{1cm}} \dots\dots\dots 0 \\ 2 \) \ \underline{5} \ \underline{\hspace{1cm}} \dots\dots\dots 1 \\ 2 \) \ \underline{2} \ \underline{\hspace{1cm}} \dots\dots\dots 1 \\ \quad \quad 1 \ \underline{\hspace{1cm}} \dots\dots\dots 0 \end{array} $	$ \begin{array}{r} 2 \) \ \underline{64 + 16 + 8 + 1} \ \underline{\hspace{1cm}} \\ 2 \) \ \underline{32 + 8 + 4} \ \underline{\hspace{1cm}} \dots\dots\dots 1 \\ 2 \) \ \underline{16 + 4 + 2} \ \underline{\hspace{1cm}} \dots\dots\dots 0 \\ 2 \) \ \underline{8 + 2 + 1} \ \underline{\hspace{1cm}} \dots\dots\dots 0 \\ 2 \) \ \underline{4 + 1} \ \underline{\hspace{1cm}} \dots\dots\dots 1 \\ 2 \) \ \underline{2} \ \underline{\hspace{1cm}} \dots\dots\dots 1 \\ \quad \quad 1 \ \underline{\hspace{1cm}} \dots\dots\dots 0 \end{array} $
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$$\therefore 89_{10} = 1011001_2 .$$

The short-division algorithm is mechanical but not easy to understand its theory behind. My method is straightforward but a bit slow. Students are allowed to use either method in tests and examinations.

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