

# An Euler Study Tour

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The Mathematical Association of America organized an Euler Study Tour that took place on July 1 to 14, 2007, in commemoration of the 300<sup>th</sup> anniversary of the great Swiss mathematician Leonhard Euler (1707 – 1783). As Euler is my ‘hero’ I seized upon this opportunity to join the tour, which turned out to be a very interesting and worthwhile experience.

For me the most worthwhile part is the opportunity to read the many original manuscripts (books, letters, notebooks) from the pen of Euler, now kept in the archives in St. Petersburg and in Berlin. The neat and beautiful handwriting, the meticulous and copious account and the substantial content enhance my admiration and respect of this great mathematician and personality. While I walked on the spiral stone staircase and the floor of the place that the great man once worked in (then the St. Petersburg Academy and now the Science Museum of Kunstkamera) and looked at the instruments that he once used, the feeling was both strange and uplifting. In Berlin we visited the Prussian Academy (now the Berlin-Brandenburg Academy of Sciences and Humanities) where Euler once worked in and the house where he once lived in. We also visited Basel where he studied as a boy in school and as a young student under the tutelage of Johann Bernoulli.

I was fortunate enough to have the honour to deliver a eulogy on the great man before his tomb in the Alexander Nevsky Monastery Cemetery in St. Petersburg on July 3. The text of the eulogy is attached below.

## **Eulogy of Leonhard Euler (1707 – 1783)**

*Respectfully read before the tomb of Euler at the Alexander Nevsky Monastery Cemetery in St. Petersburg on July 3, 2007.*

To commemorate the three hundredth anniversary of the great Swiss mathematician Leonhard Euler we would have to transport ourselves to the Europe in the eighteenth century, which can be justly hailed as ‘the Age of Euler’. As the late Clifford Truesdell, himself a renowned mathematician and a life-long admirer of Euler, put it, “To study the work of Euler is to survey all the scientific life, and much of the intellectual life generally, of the central half of the eighteenth century.”<sup>1</sup> It was a time when academic activities and scientific progress in Europe took place most prominently in the few royal academies rather than in the many universities. It was also a time when no national distinction or geographical boundary was recognized in the pursuit of science, so that eminent scholars came from different parts of Europe to study and work side by side at the same academy. Among them was the “incomparable Leonhard Euler” (in the words of Johann Bernoulli).

Euler left his homeland at the age of twenty and spent the next fifty-six incredibly productive years in the Russian Academy in St. Petersburg and the Prussian Academy in Berlin. The French mathematician-physicist François Arago said of him, “Euler calculated without apparent effort, as men breathe, or as eagles sustain themselves in the wind.” The French philosopher-mathematician Marquis de Condorcet said in a eulogy of Euler, “He ceased to calculate and to breathe.” That happened on the evening of September 18, 1783 when Euler suffered from a stroke at his home in St. Petersburg. Today we come in homage to his burial ground.

Euler is well known for his prolific output of an order of magnitude unparalleled in history, both in quality and quantity. There is this amusing story that the accumulation of the stack of his finished memoirs increased more rapidly than the rate these memoirs went to press, so that very often they were published in the reversed order of completion with the strange consequence that the content of a published memoir was more extended and improved than that

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1 C. Truesdell, Leonhard Euler, Supreme Geometer, in C. Truesdell, *An Idiot's Fugitive Essays on Science*, Springer-Verlag, New York, 1984, 337 – 379.

of another that appeared later!

Approximately one third of the research in mathematics, mathematical physics and engineering mechanics published in the last three-quarters of the eighteenth century was authored by Euler. The modern revision of his collected works (*Opera Omnia*) began in 1911 and is not yet finished, with already near to eighty volumes published by 1994. In 1983 a special issue of *Mathematics Magazine* dedicated to his memory listed forty-four items of mathematical terms or theorems that bear his name. His versatility as an all-round researcher is phenomenal, both in the continuous and in the discrete, so that he was truly an expert in “con-crete” mathematics!<sup>2</sup> The influence of Euler goes far beyond the eighteenth century and into the era we live in. Just cite two examples. His idea on the famous seven bridges of Königsberg led to the Guan-Edmonds algorithm on the Chinese Postman Problem in the early 1960s. It also led to the efficient Christofides algorithm in the mid 1970s that yields a solution to the (Euclidean) Travelling Salesman Problem no worse than three-halves of the optimal solution. The second example is in number theory. Euler calculated the famous infinite sum of the reciprocal of consecutive squares, also known as  $\zeta(2)$ , to be  $\pi$  squared over six, and treated successfully  $\zeta(n)$  for  $n$  even. Only more than two hundred and forty years later did Roger Apéry prove that  $\zeta(3)$  is irrational, but nobody yet knows what  $\zeta(3)$  is. In recent years both interest and progress are rekindled in the investigation of  $\zeta(n)$  for odd  $n$ .

As an author and teacher Euler is noted for his clarity in exposition, but more than that he was eager to share with his readers his ideas that led to those discoveries.<sup>3</sup> No wonder Pierre-Simon Laplace said of him, “Read Euler, read Euler. He is the master of us all.” His primary interest was in the wonder of

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2 *Concrete Mathematics* is the title of the book written by R.L. Graham, D.E. Knuth, O. Patashnik, published by Addison-Wesley in 1989. The authors say that they are not bold enough to try “distinuous mathematics”!

3 Many more examples, besides the computation of  $\zeta(2)$ , can be found in M.K. Siu, Euler and heuristic reasoning; Mathematical thinking and history of mathematics, in *Learn from the Masters! Proceedings of the Kristiansand Conference on History of Mathematics and its Place in Teaching*, August 1988, edited by F. Swetz et al, Mathematical Association of America, Washington D.C., 1995, 256 – 275; 279 – 282.

discovery and its explication, but cared little whether he or somebody else made the discovery. The late Russian historian of mathematics, Adolf P. Youschkevitch, described Euler as a well disposed man not given to envy, and further borrowed from the famous eulogy by Bernard le Bouyer de Fontenelle on Gottfried Wilhelm Leibniz to highlight this noble trait of Euler, “He was glad to observe the flowering in other gardens of plants whose seeds he provided.”<sup>4</sup>

One more thing about the man that earns our admiration is the way he led a fruitful life despite adversity. He was seriously ill at twenty-eight that led to loss of sight in his right eye at thirty-one, then was struck by illness again at fifty-nine that led to partial loss of sight in his left eye, and at sixty-four became totally blind for the last twelve years of his life. At sixty-four he was further hit by misfortune in losing his house and properties in a fire, followed two years later by the death of his first wife. Euler had thirteen children, but eight died in infancy. He took all the misfortune in stride and was so dedicated to his work that almost half of his research output was produced after he was near to sixty.

In the ancient Chinese Book of Odes one reads the verses that have come to connote the noble character we admire and the virtuous deed we emulate : “高山仰止，景行行止 (The high mountain I look up at it. The great road I travel on it.)” Euler was a genius, whose height very few can hope to reach. But we can all learn from his great zest for life, work and study, his insatiable curiosity to know and to probe, his determination to procure deeper and deeper understanding, his industry, his modesty, his generosity, and his toughness in facing adversity with tranquility. These are qualities we try to strive for.

The inscription on a plaque put up in his memory in his hometown (Riehen near Basel) sums up succinctly the life of this simple yet great man, “Er war ein grosser Gelehrter und ein gütiger Mensch (He was a great scholar and a kind man).”

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<sup>4</sup> A.P. Youschkevitch, Euler, in *Biographical Dictionary of Mathematicians*, Volume 2 (from *Dictionary of Scientific Biography*, Volume 4), edited by C.C. Gillispie, Scriber’s, New York, 1970 – 80, 736 – 753.