Interactive Geometric Modelling in Virtual Space

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1. Introduction

Modelling is a central term of "Mathematical Literacy" within the scope of the PISA-Study (OECD 1999). The total action of solving an applied task is referred to as the process of modelling. The following diagram shows the modelling process which proceeds in cycles. If a relevant result is validated, the modelling process ends.

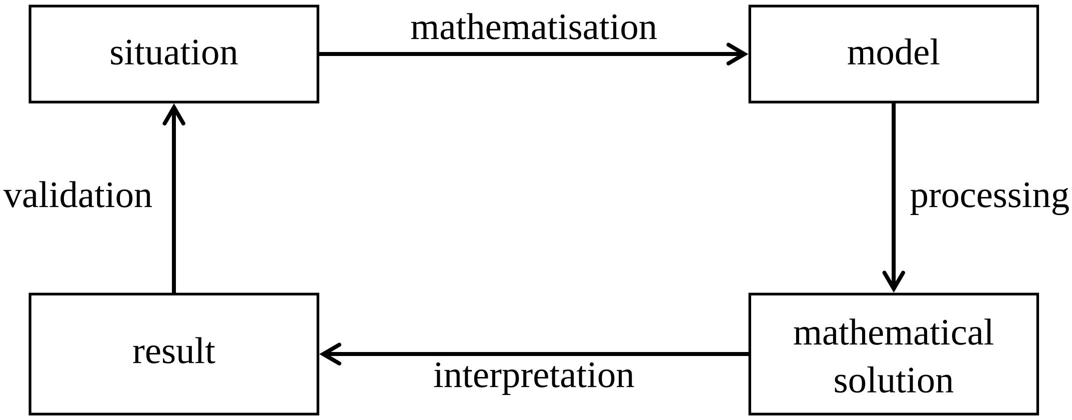


Diagram 1 (Modelling process of PISA-tasks)

In the curriculum framework of mathematics for secondary school level "modelling" also belongs to the general mathematical competences.

The respective explanations of the modelling process in the context of mathematics in school do not distinguish between modelling as the reconstruction of things already existing and modelling in terms of designing new things.

In geometry class it is comparatively referred to as reconstructive modelling (cf. Schumann 2004) and "innovative modelling" in the manner of designing new geometric configurations. (Today geometric designing is an immanent process in almost all product planning. This process proceeds computer aided by means of sectoral CAD-systems. Naive users apply for instance primitive house designing and furnishing programmes.) Therefore, both aspects of modelling should attract interest if interactive tool software is used in geometry class, above all they cannot be distinguished strictly.

A reconstructive task of modelling within the scope of spatial geometry class consists of the reconstruction of the respective spatial geometry, which has been implemented into a physical object during its generation; whereas, an innovative modelling task contains the construction of a physical object not yet existing (The purpose of this could be the creation of it in the scope of connecting geometry and engineering class). This can be pointed out by the following example. If a table or the picture of it is given, the task is: construct a virtual model of the table (reconstructive modelling). The corresponding task regarding innovative modelling could be: plan a table and model it in virtual space. The reconstructive task is closed concerning its objective whereas the type of modelling and the way to solve the problem are open. Furthermore, the innovative task is open in regard to its objective; its solution requires more imagination and creativity.

Interactive modelling in virtual space requiring an adequate interactive spatial geometry tool with high quality in regard to software ergonomics and spatial perception is relatively new. Cabri 3D extensively meets this claims.

With the spatial geometry system Cabri 3D version 1.1 a tool is provided that

- carries out spatial geometric constructions in the “depth” of screen by extending plane to spatial constructions or constructing directly spatially,
- visualises spatial geometric configurations by designing them with comprehensive object attributes and viewing them from different perspectives with *Virtual Sphere Device* a device that incorporates configurations into a sphere, which can directly be referred to,
- varies spatial geometric configurations by dragging as already common in plane dynamic geometry systems.

Modelling with Cabri 3D is supported by general methods, which respective operation modes have already applied and designed when using dynamic 2D geometry systems.

From our point of view, this way of using computers is an example of effective integration of computers in spatial geometry class which is little developed in regard to spatial geometric constructions.

Applying Cabri 3D in order to model/design geometrically in virtual space the following general geometric objectives are aimed at:

- teaching the geometric vision respectively "eye" and experience virtual space as a room for action (perceptional objective)
- cherishing the utility of spatial geometry (affective objective)
- extending and applying spatial geometric knowledge (terms, statements, and processes) (cognitive objective)
- analysing and exploring experimentally spatial phenomena which can be designed geometrically (meta cognitive objective)
- practising the use of Cabri 3D (technical objective)

The process of spatial modelling/designing is shown in the following diagram:

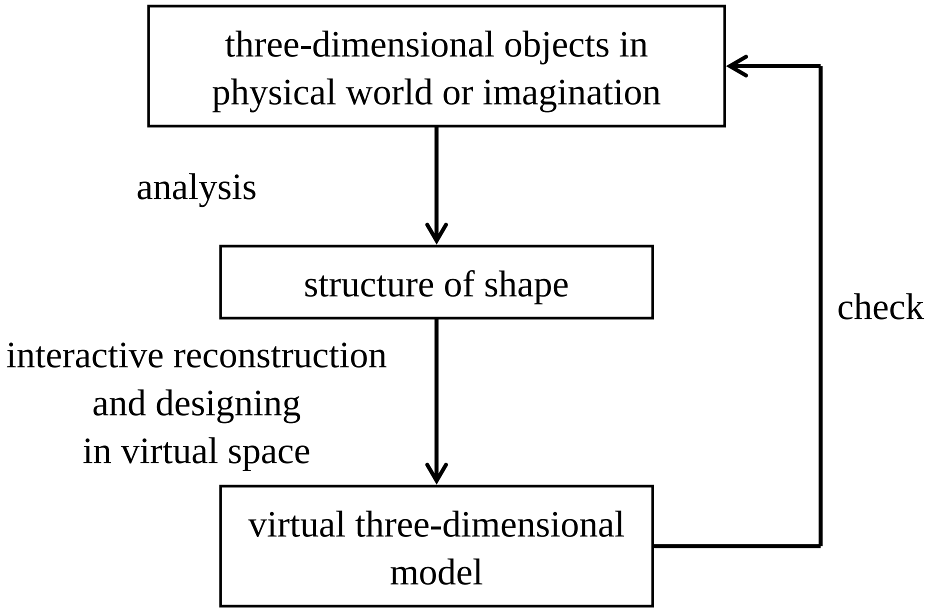


Diagram 2 (Modelling process with Cabri 3D)

The process of modelling/designing in order to achieve static models is described now as a project instruction for pupils:

Instructions on the creating of a model are as follows:

1. Choose a three-dimensional object from your environment (in printed version respectively from the internet) or an object of your imagination, which can be analyzed and reconstructed/designed by means of spatial geometry.
2. Explore and analyse this object in regard to special geometric shapes and principles. If applicable, get further information on the object. Draw a manual sketch, too.
3. Reconstruct/design the identified spatial shape or configuration by using tools of Cabri 3D.
4. Check the result of your reconstruction/design by comparing it to the original object/object of your imagination.
5. Publish your checked model/design together with a description and a picture of the original object/your manual sketch of the design on the internet or as a wall paper in your class room.

These modelling processes with Cabri 3D are liable to the following basic restriction: Only objects are considered to be modelled which can be described by means of spatial elementary geometry and furthermore, which can be reconstructed with the instruments and methods of Cabri 3D. A reconstructive modelling in space comparable to two-dimensional dynamic geometric systems would call for a digital survey of the spatial object (an already three-dimensional digital modelling process). On the one hand, corresponding techniques of digitalisation for use in school geometry are not available yet and on the other hand, three-dimensional school geometric tools which would allow an import of such three-dimensional pictures as patterns of modelling do not exist.

Another problem results in the consideration of cognitions concerning the object of modelling which are not considered in school mathematics. Disregarding these cognitions causes the danger of dilettante work.

2. Examples of Modelling/Designing in Virtual Space With Cabri 3D

In the following, results of modelling processes on some chosen examples with different degrees of difficulty are documented. Technical details of using Cabri 3D 1.1 are not going to be delivered here. They are mainly described in the manual which could be downloaded from www.cabri.com.

The collection of examples is supposed to animate own actions. In this context, the consulting of pupils by competent teachers in regard to the adequacy of the objects of the physical world to be reconstructed or the objects part of the mental world to be designed is necessary.

The modelling with any polygons, which could be generated as referable objects, predominates in Cabri 3D 1.1. Some regular convex polygons and star polygons are implemented as plane shapes. It is only possible to construct with the following spatial modules of convex shapes: cuboid, convex prisms, convex pyramids, all the platonic solids, convex polyhedra (as convex hull of finite points), spheres and lateral surface of cylinders and cones. If corresponding points as vertices of polygons are available or can be constructed, it is quasi possible to approximate every adequate spatial surface with polygons, particularly with triangles. If the object of modelling or designing is symmetric, the polygonal approximation without modules of shapes is notably simple. Spatial constructions are supported by doubling configurations with the options of spatial congruence mappings. – In addition to the compasses and straightedge constructions in any plane one can execute any construction in space using the “straightplane” and “spherical compasses” tool. Unfortunately the definition of construction macros is not possible yet.

The variation of measurements of dimensioning is a characteristic of dynamic modelling/designing with Cabri 3D: the shape of the model can be varied by dragging basic points.

In the following, the focus is not on surface design of the models, but on the acquisition and the constructive realisation of a spatial geometric structure of the objects to be modelled/designed.

2.1 Roofs

Roofs are popular, application oriented, spatial geometric objects, which can be used in order to practise examination of shapes (e.g. identification of symmetry characteristics), calculations, and figures. Roofs are each designed in a cuboid frame which can be dimensioned optional in width, length, and height (Figures 1.1 – 1.6 types of rooftops; Figures 1.7 – 1.9. types pyramidal broach roofs). By dragging parameter points of the roof, roofs can be transformed to border cases of other roofs; therefore, tent roof or gabled roof are border cases of the hipped roof, for instance.

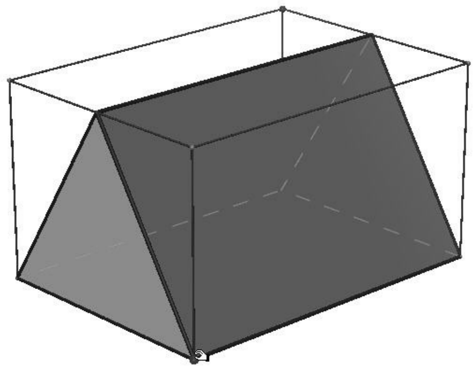


Figure 1.1: Gabled roof with frame of construction

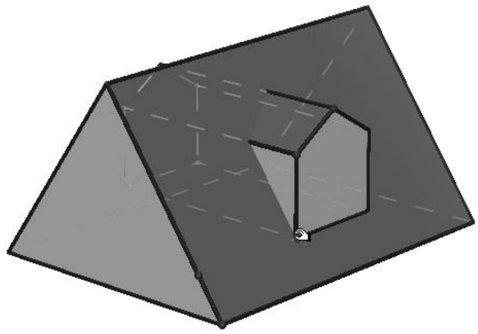


Figure 1.2: Gabled roof with dormer

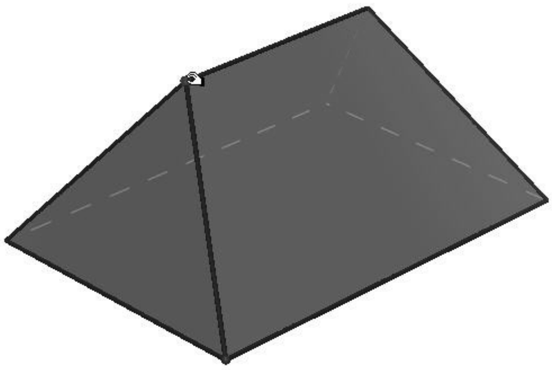


Figure 1.3: Hipped roof

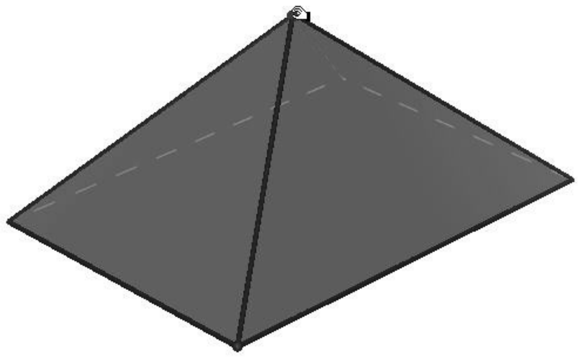


Figure 1.4: Tent roof

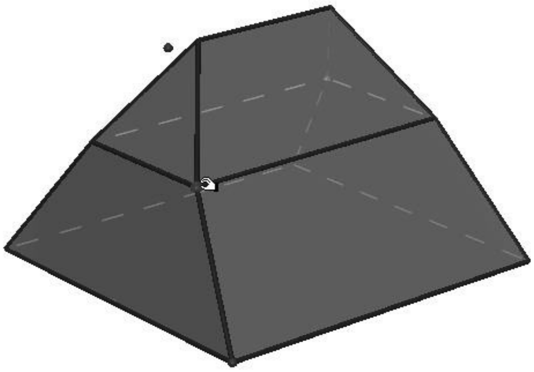


Figure 1.5: Half-hipped roof

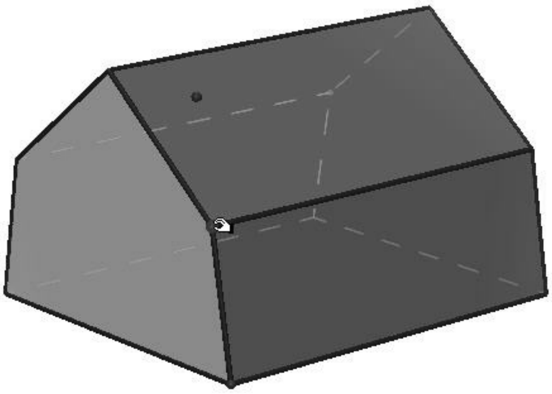


Figure 1.6: Mansard roof

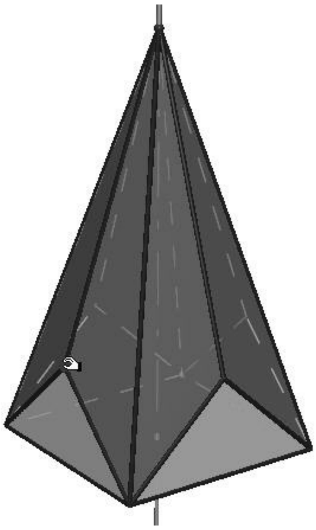


Figure 1.6

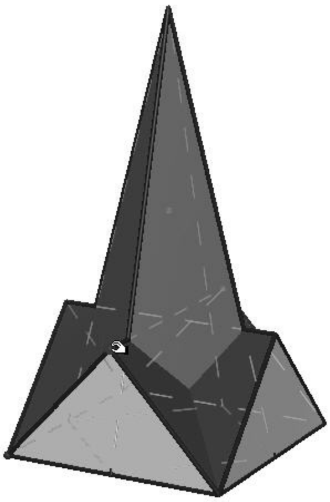


Figure 1.7

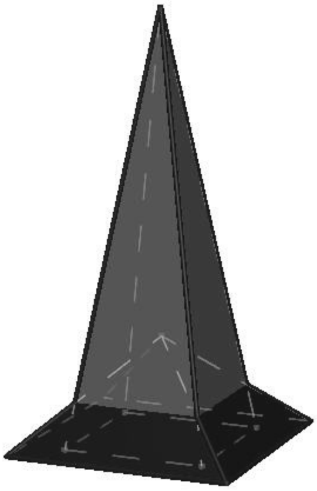


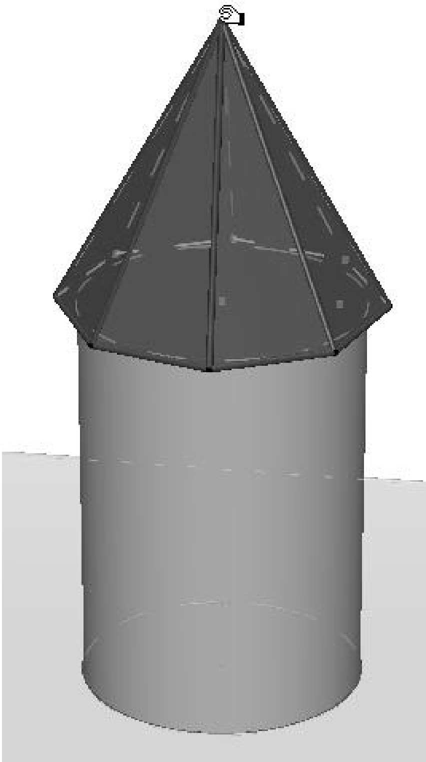
Figure 1.8

Different pyramidal broach roofs

2.2 Buildings

Modelling of buildings depends in the first place on recognizing the modules of shapes like cube, cuboid, prism, cylinder, cone, sphere – shapes of which the building can be composed of.

Modelling the towers in figures 2.1 – 2.3, the “Gruene Turm” in Ravensburg (the city of the games with the blue corner), the “Berlin television tower” and the “exhibition tower” in Frankfurt am Main acted as a model. – There are lots of skyscrapes or buildings of Hong Kong suitable to be reconstructed three-dimensional in their outlined geometric shape. In addition one can reconstruct a whole gathering of buildings of the famous skyline of Hong Kong, which is a challenging project task.



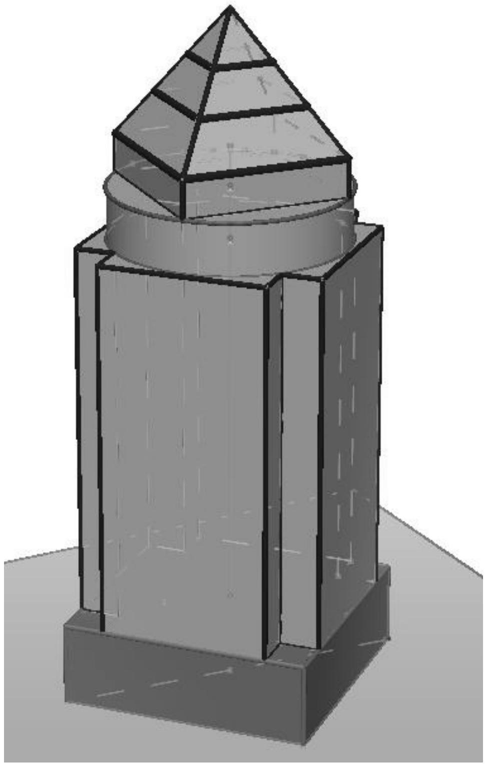


Figure 2.1 Figure 2.2 Figure 2.3

Different towers (in different scales)

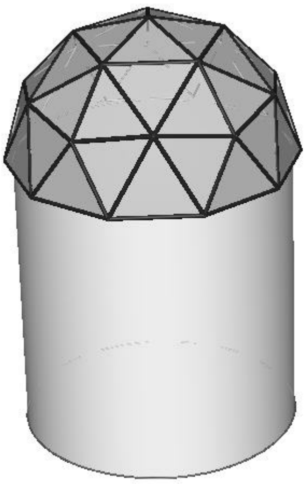


Figure 2.4:
Tower with geodetical cupola

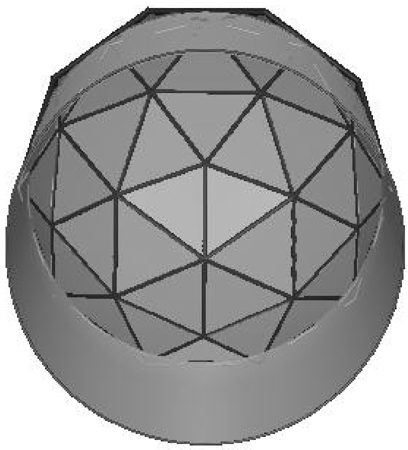


Figure 2.5:
Cupola seen from the bottom

The construction of a geodetical cupola (Figure 2.4 as pyramidal broach roof) based on regular icosahedron is a challenging task. Figure 2.5 gives an impression how such a cupola looks like from the bottom.

Different shapes of roofs can be completed to buildings by adding a cuboid or several cuboid modules as main body (Figures 2.6 – 2.8).

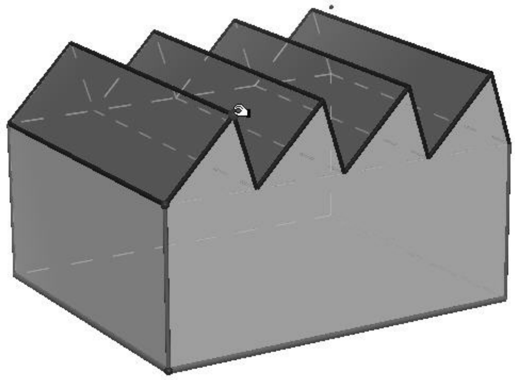


Figure 2.6: Factory building with structure of pent roof

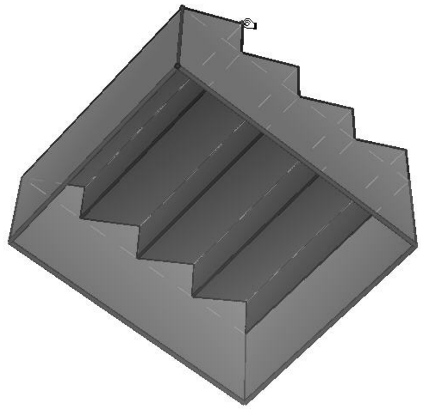


Figure 2.7: Factory building seen from the bottom

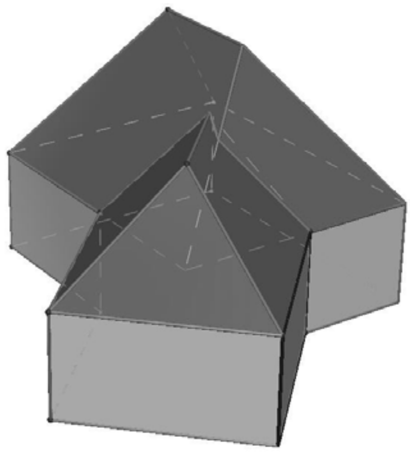


Figure 2.8: Angle house with intersection of hipped roof

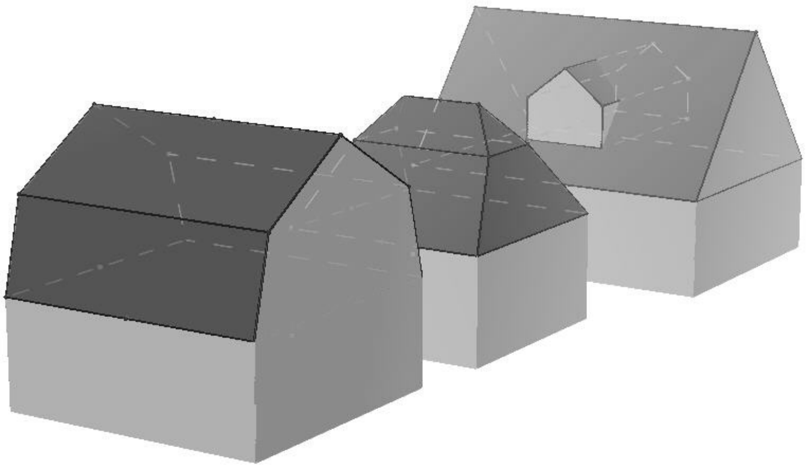


Figure 2.9: Three houses with different shaped roofs in a row

Single models can be combined to groups of models by copy and paste. This is shown in Figure 2.9 with three houses. The fading of the objects in the foreground to the background amplifies the impression of spatial depth.

Modelling complex parts of a structure like a Romanic dome is a time-consuming project task (Figures 2.10 – 2.12). The basic model in this context is the cube. The height of the ridging, the heights of the pyramidal broach roofs, and the height of the twin towers can be varied by dragging. The dome can be alienated by dragging, e.g. the dome in outlines or with unrealistic modified roofs (Figures 2.13 – 14). This could be used when teaching arts in order to make pupils proportionate the dome correctly.

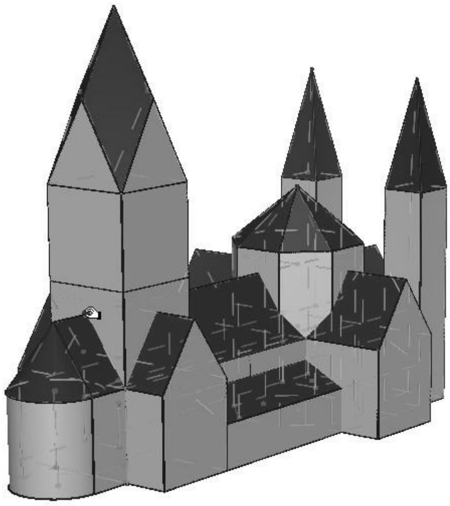


Figure 2.10: Romanic dome (from northeast)

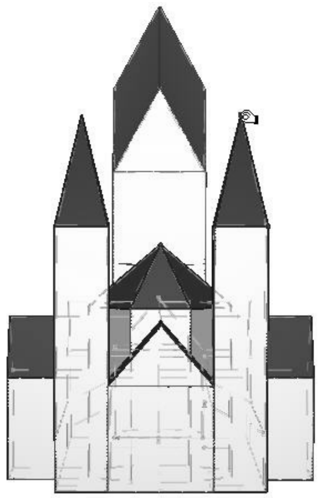


Figure 2.11: Romanic dome (from west)

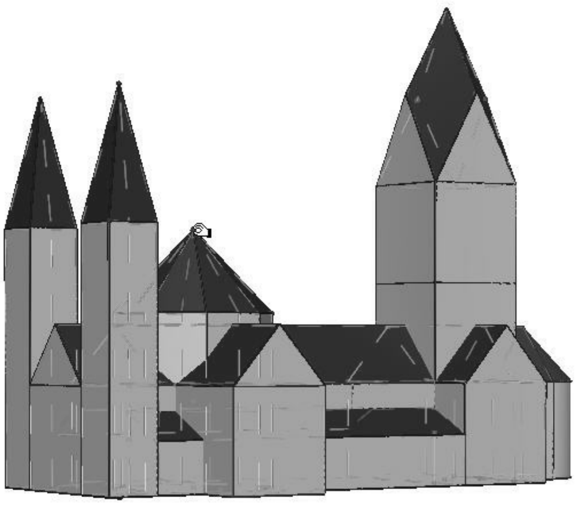


Figure 2.12: Romanic dome (from southwest)

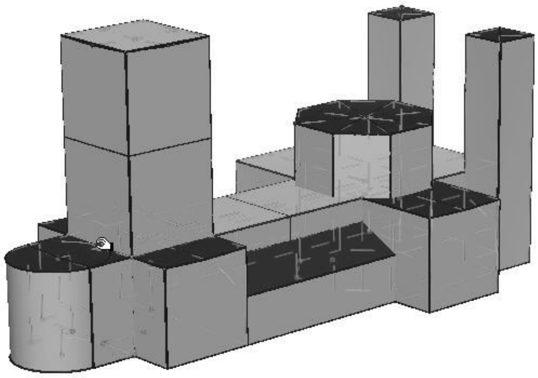


Figure 2.13: Alienated Romanic dome (in outlines like a ruin)

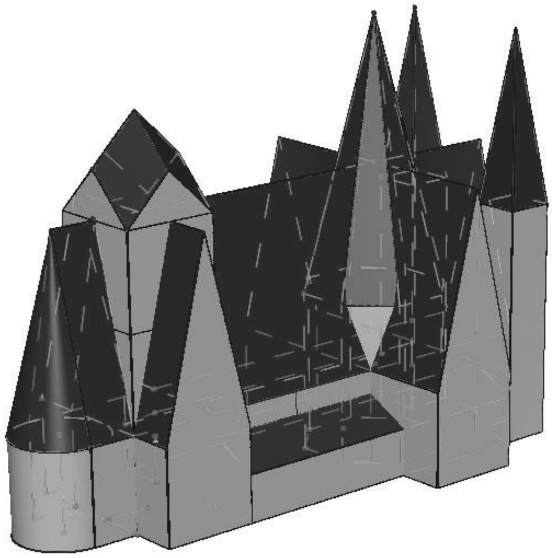


Figure 2.14: Alienated Romanic dome (with unrealistic modified roofs)

Of course, curious buildings like the “Atomium” and that of the EU-Council in Brussels can be modelled as well (figures 2.15 – 16).

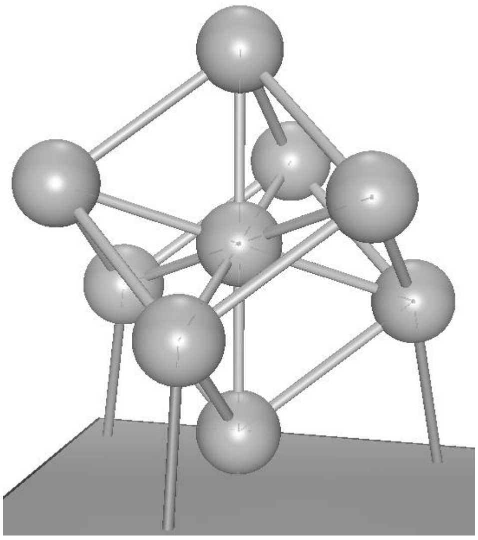


Figure 2.15: Model of the “Atomium” (World exhibition 1958 in Brussels)

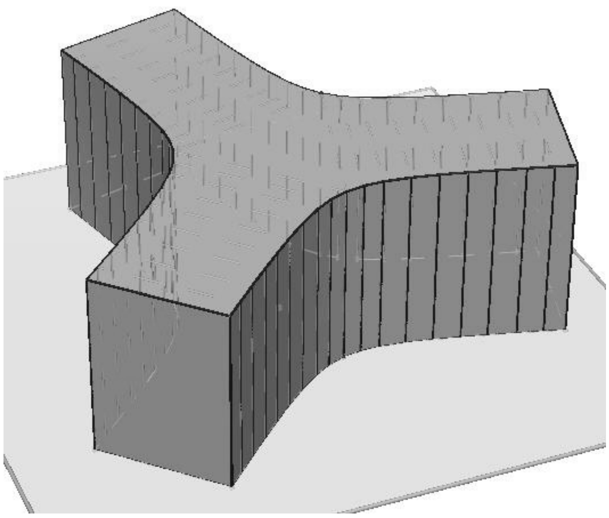


Figure 2.16: A Variation of the EU-Council Building in Brussels

2.3 Lamps and Shiners

Lamps and shiners can easily be modelled with Cabri 3D.

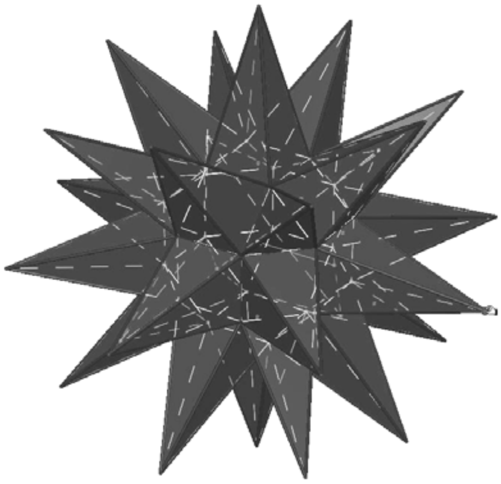


Figure 3.1: Christmas star (a particular stellation of a cuboctahedron)

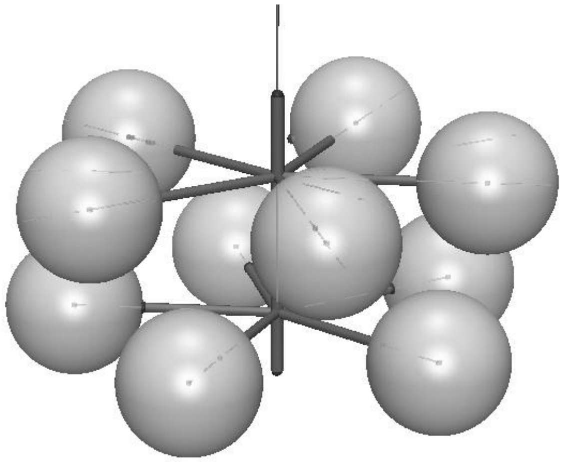


Figure 3.2: Lamp consisting of spheres (living room lamp)

Figure 3.3: Standard lamp (lampshade is the lateral area of a frustum of pyramid)

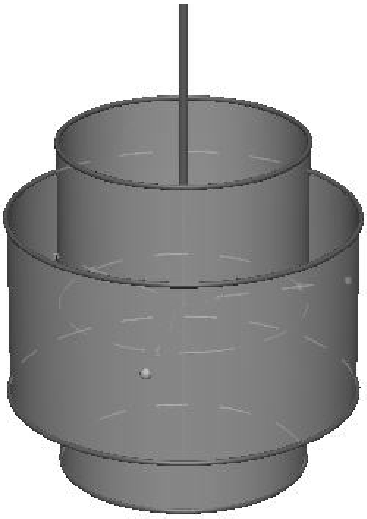


Figure 3.4: Drop light

Figure 3.5: Lantern

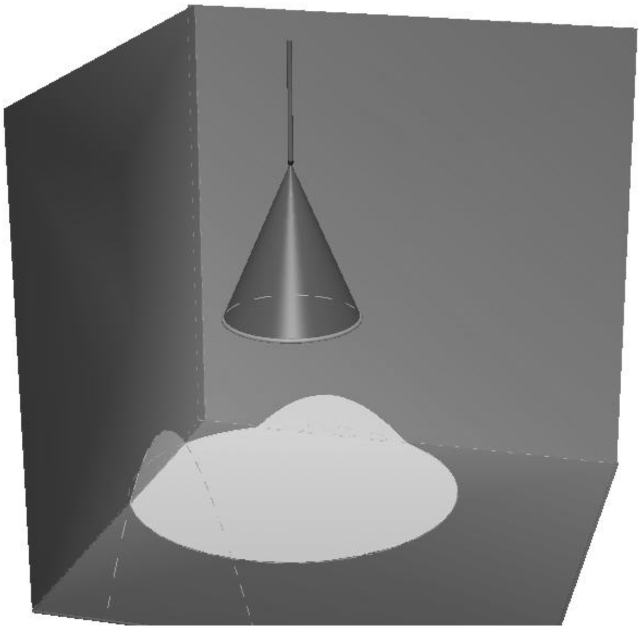


Figure 3.6 Lamp with cone of light

2.4 Miscellaneous

The advanced variety of modelling/designing possibilities can only be adumbrated by the following results of modelling/designing.

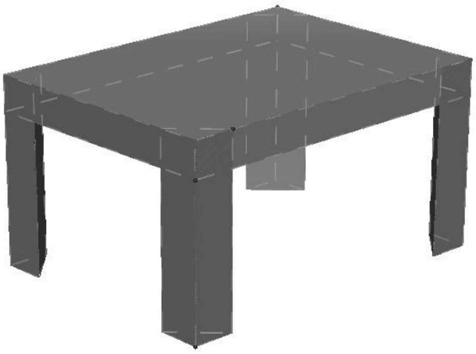


Figure 4.1: Table

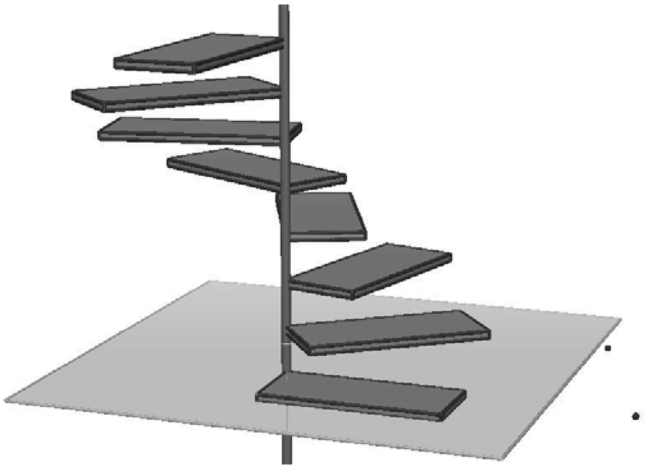


Figure 4.2: Spiral staircase (with a turnaround of 270°)

Figure 4.3: Circular brilliant cut with non-regular optical path

Figure 4.4: Circular brilliant cut with regular optical path

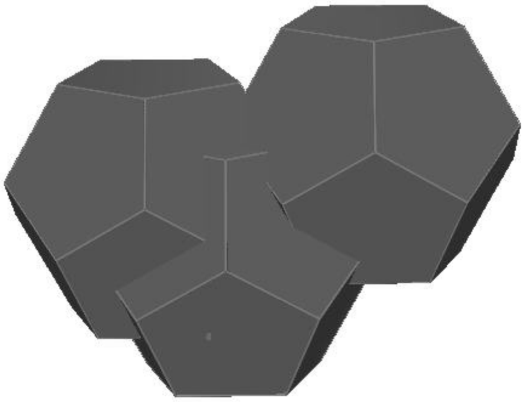


Figure 4.5: Crystal-agglomeration consisting of regular dodecahedron (triple of a Pyrit crystal)

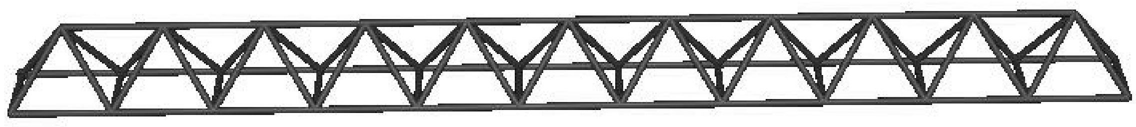


Figure 4.6: Structure consisting of models of edges of regular tetrahedrons and square pyramids with same edges used in buildings as carrying element for cranes as load arm

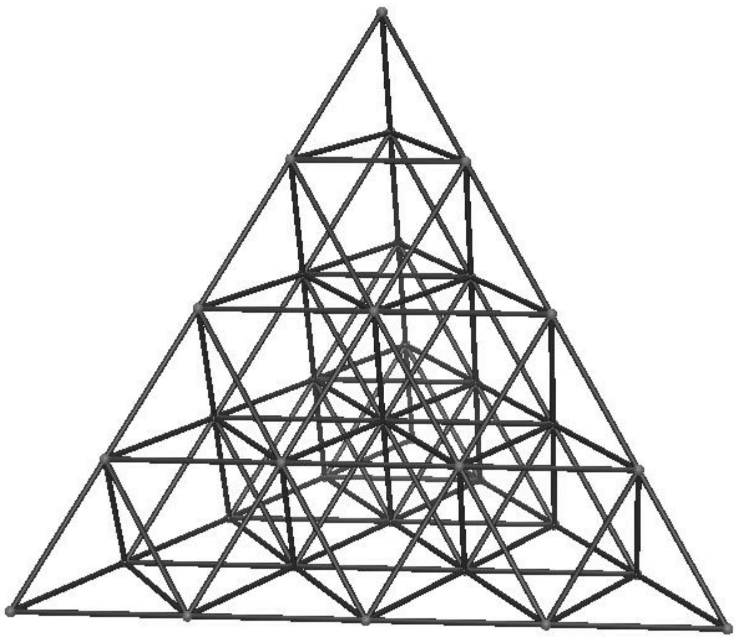


Figure 4.7: Pyramidal structure consisting of models of edges of regular tetrahedron, e.g. for climbing in a playground

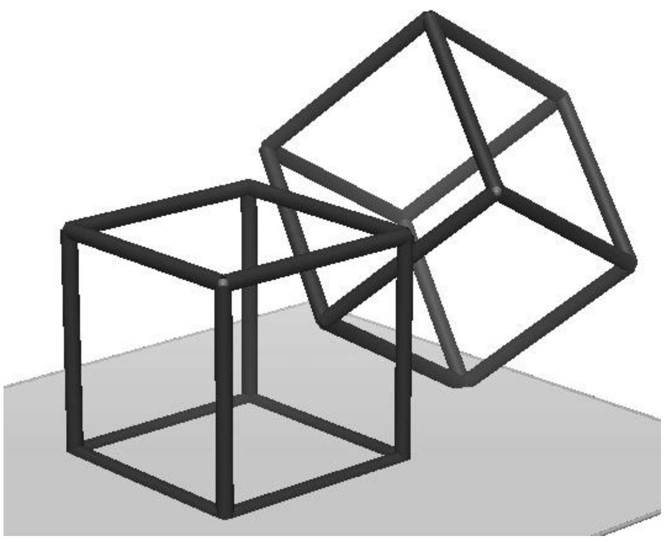


Figure 4.8: Based on a plastic of the German sculptor Alf Lechner

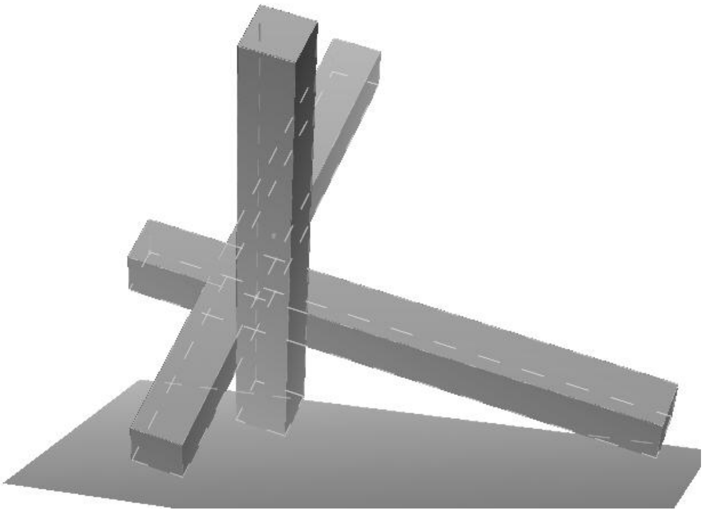


Figure 4.9: Plastic consisting of three quadratic columns – imitated by Max Bill, a famous representative of Constructivism

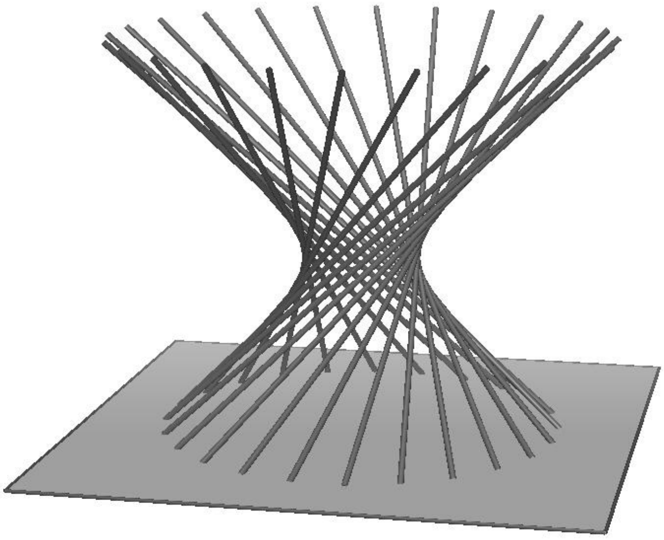


Figure 4.10: (Mikado-) sticks, creating a rotation-hyperboloid

3. Literature

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