Three Solutions for a CE Multiple-choice Question

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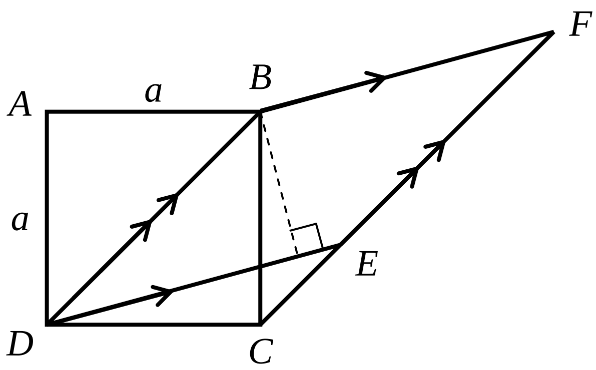
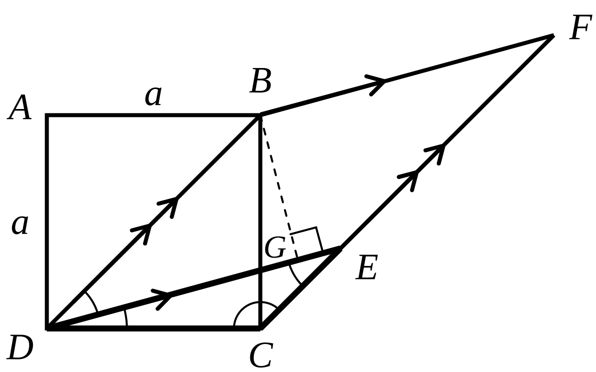
Recently, my students from different classes (including students from junior forms) asked me a CE multiple-choice question. Here is the question and three different solutions.

Question (92-CE-Maths II #54)

In the figure, $ABCD$ is a square of side a and $BDEF$ is a rhombus. CEF is a straight line. Find the length of the perpendicular from B to DE.



Solution 1



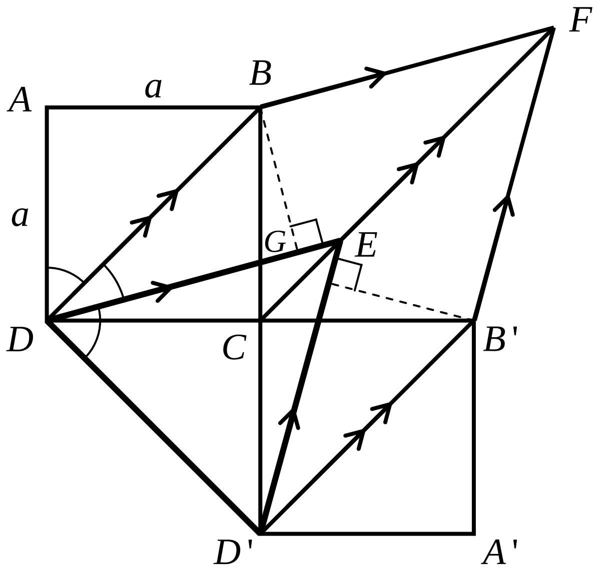
$DE = DB = \sqrt{2}a$, $DC = a$, $\angle DCE = 135°$.

Consider ΔDCE and using the sine formula, we have $\angle DEC = 30°$.

Hence, $\angle CDE = 15°$, $\angle BDG = 30°$.

Consider ΔDBC , we have $BG = \dfrac{a}{\sqrt{2}}$.

Solution 2



Reflect the figure along CF .

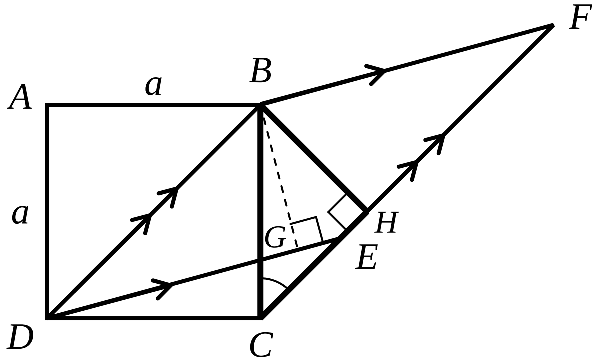
$\Delta DED'$ is an equilateral triangle, i.e. $\angle EDD' = 60°$.

$\angle ADB + \angle BDE + \angle EDD' = 135°$ and $\angle ADB = 45°$.

Therefore, $\angle BDG = 30°$.

$BG = \dfrac{a}{\sqrt{2}}$.

Solution 3



$\angle BCH = 45°$.

Therefore ΔBCH is an isosceles triangle.

$BH = \dfrac{a}{\sqrt{2}}$.

$BG = BH = \dfrac{a}{\sqrt{2}}$.

The initiative ideas of Solution 1 and Solution 2 are simliar and straight-forward. If we know the measure of $\angle BDG$, we can find BG . Solution 1 uses sine formula which is taught in senior form. Solution 2 does not need to use sine formula, but students seldom come across such kind of solution.

Solution 3 is creative and it is suggested by a junior form student. Instead of finding the measure of $\angle BDG$, the student notices that $BG = BH$. Students can find BH with sine. However, since ΔBCH is an isosceles triangle, students can find BH with the Pythagoras Theorem. That means students can solve this problem with simple geometrical knowledge only!