Empowering Student Thinking in Learning Mathematics by Effective Questioning

TANG Kon-ming
The Chinese University of Hong Kong

[I] Introduction

Mathematics is a mode of thinking, a powerful means of communication, a tool for studying other disciplines and an intellectual endeavour (Curriculum Development Council, 2002). However, could these characteristics of Mathematics be demonstrated in our classroom learning? It was found that students in Hong Kong spend over one-third of their homework time on mathematics homework (Wong et al., 1999). They need to do lots of computations and routine problems in order to familiarize themselves with the algorithmic procedures. The only objective is to avoid failure in the examination and succeeded in obtain a place in the next stage of study. Examination orientation reinforces learning by rote (Wong et al., 1999).

The situation remains unchanged though the universal education was implemented in the late 70's because of the competitive nature of education in Hong Kong. As a result, while students think that understanding is important in learning mathematics, getting the correct answers quickly is deemed even more important (Wong et al., 1999). Although Hong Kong students perform exceptionally well in international mathematics studies, Hong Kong public examinations reveal that the major problems for students are their inadequacies in tackling problems and in their mental processes, rather than a lack of skills or basic knowledge (Wong et al., 1999). In other words, fosterling of students thinking is inadequate and being disregarded.

In order to emphasize the importance of students thinking in learning mathematics, teachers should revitalize the significance and function of their classroom questioning. As Beyer (1997) pointed out that questioning has been one of the most common techniques used over the years to teach thinking.

However, one of the most commonly used methods in elementary and secondary school teaching is recitation, which involves rapid question-and-answer exchanges orchestrated by the teacher, usually for the purpose of assessing how well students have mastered the content of a lesson (Wilen, 1992). Therefore, only skillful questioning of student thinking can provide the teacher with essential knowledge about students' developing mathematical ideas, knowledge which might be otherwise inaccessible (Martino & Maher, 1999). In other words, teachers should equipped with a systematic and logical questioning sequence which could sharpen students' perceptions, refine their thinking and connect the unknown to the known as advocated by Dantonio and Beisenherz (Dantonio & Beisenherz, 2001, p.53).

In this article, the functions of questioning is reviewed and a framework of an instructional strategy proposed by Dantonio & Beisenherz (2001), "Questioning for understanding: Empowering student thinking (Qu:Est)" is introduced. The instructional strategies focus on individual thinking operations necessary for constructing concepts. They also assist teachers in asking productive questions that facilitate students' conceptual awareness and understanding. In learning mathematics, conceptual understanding is crucial because mathematics is much more abstract than any of the other subjects which children are taught at the same age, and this leads to special difficulties of communication (Skemp, 1989). Furthermore, the instructional strategies provide a substantial framework as the Qu:Est was developed using principles of learning derived from the literature on effective questioning practices, teaching for understanding, and how children learn (Dantonio & Beisenherz, 2001, p.52).

In order to have an explicit implementation of the instructional strategies, two local topics, each from the primary and secondary mathematics curriculum are selected to be the exemplars. The two exemplars also demonstrate how teachers provide opportunities for pupils to use mathematical language, such as explaining results and briefly describing the methods used in investigating problems in oral form with the help of diagrams as suggested by the Curriculum Development Council (Curriculum Development Council, 2000, p.49)

[II] Functions of questioning

An effective learning and teaching strategies always request teachers to change their role from solely knowledge transmitters to all-round educators, such as facilitators, consultants, counselors, and assessors. No matter which role teachers are playing, communication or interaction between teachers and their students is inevitable. Teachers can give students opportunities to express themselves openly and share their work in class and publicly to enhance their confidence (Curriculum Development Council, 2001). In addition, the Curriculum Development Council (2001) also pointed out that since public feedback indicated that the curriculum reform envisage is too broad, a priority focus will be placed on the development of three of the nine generic skills, namely communication skills, creativity and critical thinking skills (p.25). In fact, the nurture of any generic skills, values and attitudes is derived from the learner's own thinking (Tang, 2002). Hence, how teachers could succeed in developing students' generic skills should be seen as something of first priority. One of the ways is to promote teachers' effective questioning technique. In what follows, four functions of questioning will be discussed accordingly.

(A) Elicit Thinking

Questioning is an important part of the teacher's ability to establish a classroom atmosphere conducive to the development of mathematical thinking (Burns, 1985). Besides, teachers' questions can stimulate thinking, facilitate class discussions, evoke expression and probe thought process as well (Dillon, 1982; Wilen, 1992). It is especially significant to the young students whose mental activities are highly dependent. Wilen (1992) pointed out that a question can arouse curiosity and stimulate mental activity. In responding teachers' questions, students must make use of their thinking operations, such as comparing, contrasting or grouping etc. Once students have given their responses, Bulgar et al. (2002) suggested to use responsive questioning to elicit explanations, to help students develop appropriate justifications and to redirect them when they were engaged in faulty reasoning. Or to help the students to examine their own and each other's ideas. All these processes elicit students' thinking. And elicit depth of processing that is meaningful to the students

(Wilen, 1992). Therefore, questioning is a useful means to clarify and expand thinking (Sund & Carin, 1978). Questions can become a catalyst for urging learners to justify their ideas and explain them to each other. This, in turn, has the effect fostering deeper thinking about the ideas involved in the problem situations (Bulgar et al., 2002).

(B) Facilitate communication

Passive learning is always perceived as ineffective. It is unimaginable to have the teacher merely talking inside a classroom. Through questioning, teachers may communicate the elements of the subject matter with their students. By responding teachers' questions, students have to raise their views, organize their expressions, show their learning and tune their logical thinking. In addition, by ideas sharing, students may learn from their peer. Martino & Maher (1999) advocated to allow students to play more active roles in their own and each other's learning, and thus build a classroom community that invites active participation, confidence, and further learning. Wilen (1992) highlighted that to draw attention to affective responses, such as, feelings, attitudes, appreciations, interests and values so that clarification of them will give more personal meaning to all learning. The communication of these affective responses is bridged by the teachers' timely and appropriate questions. That's why Hunkins (1976) pointed out that to probe the interests and feelings toward a phenomenon that might be identified through experience and lower-cognitive level questions.

(C) Strengthen conceptualization

If we as teachers are to find out what our students already know so that we can help them use that understanding to construct new knowledge, we need to focus on questions that will assist us in achieving that goal (Vace, 1993). It is just the first step in helping students conceptualize the new learning by identify their prior knowledge by posing recall questions. During the whole learning process, teachers have to redirect the questions for more responses, to prompt questions for successful responses, and to probe questions for true understanding. Recall responding should be considered a springboard for higher forms of understanding, rather than as an end product of learning (Ryan, 1971).

By questioning, teachers may evaluate student preparedness, support conceptual development, reinforce understanding and ask student to elaborate (Wilen, 1992). Furthermore, affective learning is as vital as cognitive one. Hence, Wilen (1992) also suggested that teachers' questions could help students work through the internalization of values and conceptualization of a value system, i.e. help students clarify how strongly s / he believes in the value.

(D) Assess learning

It is common practice for teachers to assess their students' learning by asking them questions formally or informally. And diagnosis of the mastery level of the students from their response to the questions (Wilen, 1992). Hence, one of the purposes of teacher's questions is to help teacher assess students' learning. However, the degree of concern between oral questions in the classroom context and written questions in the examination context is varied. For the teachers, they put greater effort in constructing the examination questions while classroom questions are not always well prepared in advance. For the students, they will treat the examination questions more serious than the classroom questions as the examinations are high stake activity in most of their learning process. Thus, classroom questions are always used for formative assessment and the significance of teachers' questioning inside the classroom should be emphasized.

[III] Questioning for understanding: Empowering student thinking (Qu: Est)

(A) Introduction

Learning requires students to possess a heightened awareness of various types of thinking operations and to be conscious of the role various types of thinking operations play in the conceptualization process (Dantonio & Beisenherz, 2001). However, focus on the familiarization of algorithms, immersing in close end problems, failure to accept alternative solution strategies and representation formats distort our students learning in mathematics. Students are conditioned to memorize what they are taught. Such matter of rote memorizing in learning is an obstacle in developing student thinking which in turn affect the conceptual understanding. In fact, understanding can satisfy personal needs, facilitate learning, enable flexible learning and enhance retention of knowledge (Newton, 2000).

Dantonio & Beisenherz (2001) contended that conceptualizing demands that students be in full charge of their thinking and be able to monitor their learning in this manner (p.54). Moreover, understanding often needs student's active, mental engagement with the topic in hand (Newton, 2000). It is the teacher's responsibility to provide opportunities for students to immerse the thinking processes and enjoy learning in this way rather than rote learning. One of the potential and frequently adopted way is teacher questioning.

Generally, questions asked by the teachers have served and continue to serve a variety of learning functions. Some provide opportunities for students to engage in higher-order thinking by posing thought-provoking situations that require student engagement in interpretation, application, decision making, reasoning, and so on (Beyer, 1997). However, to be effective questioners, not only should they understand the different functions of questioning, but also equip a questioning framework in order to develop student thinking and enhance their conceptual understanding effectively. The following is an introduction of the Qu: Est.

(B) Qu: Est instructional strategies

They are process-centered lesson designs that corporate productive questioning practices for carrying out instructional conversations to develop and refine students' cognitive abilities as they engage in instructional conversations about curricula (Dantonio & Beisenherz, 2001). It consists of three strategies and each involves different cognitive operations or thinking operations. The first one is the "collecting strategy" which demands two cognitive operations: observing and recalling. They are the primary ways in which students garner specific information that will be used to identify and distinguish the critical characteristics or attributes of concepts. Hence, the collecting strategy is the foundation for building concepts.

The second strategy is the “Bridging strategy” which demands three thinking operations: comparing, contrasting and grouping. They are integral to building and creating concepts. Having undergone comparing and contrasting, students are expected to discover relationships among previously isolated facts and identify the critical characteristics, attributes, or patterns of a concept. And finally they may group similar objects or ideas together according to their identification of critical characteristics. In this case, teachers need to ask productive questions that provide opportunities for students to connect isolated facts to form critical attributes or characteristics that are the basis of a concept.

The third strategy is the “Anchoring strategy” which demands two cognitive operations: labeling and classifying. They are instrumental in communicating and categorizing concepts formed by students. This strategy formalizes the construction of concepts, securing that students have established a meaningful link between specific facts and information and the naming of the concepts.

(C) Core questions in Qu: Est

In order to make students view learning as a process and not just a game of getting the right answer, teachers’ questions must provide the cognitive signals. Dantonio and Beisenherz referred such cognitively cued questions as core questions. They claimed that the core questions should focus, direct, and guide particular kinds of thinking operations and the content specified by the goals or objectives of the lessons.

There are three critical characteristics of the core questions: i) clear, ii) focused, and iii) open. Clear means that the language used must be understandable by the students. Focused means that the words used must be specific to the content and stipulate the cognitive operations or thinking operations. Open means that the words used should provide opportunities for multiple and diverse response from as many students as possible. The following table shows some general examples of core questions for the various thinking operations involving in conceptualizing:

Core Questions for Individual Thinking Operations in Conceptualizing

Operations	Core questions
Observing	What do you notice about …?
Recalling	What do you remember about …?
Comparing	What similarities are there between … and …?
Contrasting	What differences are there between … and …?
Grouping	In what way do these items go together?
Labeling	What can we call …?
Classifying	How can we classify …?

[IV] Two exemplars

The first exemplar adopts the topic of "Mid-point theorem" in the current secondary mathematics curriculum. The teaching materials consist of 6 figures (figure 1a to 3b), in 3 pairs, showing the measurements of lengths and angles. [See appendix I]. The core questions in each of the cognitive operations or thinking operations are as follows:

i. *Observing:*

1) What do you notice about the lengths of sides in figure 1a and 1b?
2) What do you notice about the sizes of angles in figure 1a and 1b?

ii. *Recalling:*

1) What do you remember about the different types of angles formed when a transversal cuts two parallel lines?
2) What do you remember about the conditions to be parallel lines?

iii. *Comparing:*

1) For the lengths of sides, what similarities are there between figure 1a and figure 1b?
2) For the sizes of angles, what similarities are there between figure 1a and figure 1b?

iv. *Contrasting:*

1) For the lengths of sides, what differences are there between figure 1a and figure 1b?
2) For the sizes of angles, what differences are there between figure 1a and figure 1b?

Repeat the above questions for the other pairs of figures.

v. *Grouping:*

1) In what way will you divide the 6 figures into two groups?

vi. *Labeling:*

1) What can we call the group involving the mid-points in the figures?
2) What can we call the line joining the two mid-points in the figures?

vii. *Classifying:*

1) How can we change the conditions of figure 1b to meet the requirement of the mid-point theorem?
2) Can you draw a figure showing the conditions for the mid-point theorem?
3) Can you draw a figure showing that the mid-point theorem cannot be applied to it?

The second exemplar adopts the topic of “Square and Rectangle” in the current primary curriculum. The teaching material is shown in Appendix II. The core questions in each of the cognitive operations or thinking operations are as follows:

i. *Observing:*

What do you notice about the shapes of the figures as shown?

ii. *Recalling:*

What do you remember about the characteristics of quadrilateral?

iii. *Comparing:*

What similarities are there among the figures?

iv. *Contrasting:*

What differences are there among the figures?

v. *Grouping:*

In what way will you divide the figures into two groups?

vi. *Labeling:*

What would be an appropriate name for each group?

vii. *Classifying:*

1) Can you give any example which is in the shape of square / rectangle in this classroom?
2) Is the shape of a A4 paper in square or rectangle?
3) How can we change the shape of A4 paper into a square?
4) Can you draw a square and a rectangle?

The above two exemplars are showing the core questions only. Students' responses to these core questions may be varied. Teachers need to assist them in responding successfully by giving them hints, clues or relating to their prior knowledge. Besides, teachers should keep the discussion going and monitor the awareness of the whole class. To promote higher-order thinking, teachers must request for clarification, justification or further explanation about the responses. Sometimes, teachers may use the first response to serve as a basis for other answers so that high level of interaction is established. In this case, teachers need to react spontaneously. Last but not least, teachers have to be supportive or inviting manner to avoid placing students in threatening or intimidating situations.

[V] Conclusion

Teacher questioning is a critical component of a more student-centered classroom. It is because a more student-centered classroom requires teachers who listen to the explanations of their students, probe them for justifications, encourage them to share their solutions with their peers as they work together to refine, revise and extend their solutions (Bulgar et al., 2002). The question aims to stimulate thought about what is relevant at the time it is asked (Newton, 2000). Conceptual understanding is achieved only if the students undergo deep thinking about the learning content. It is especially vital for learning mathematics, which demands 95% intelligent learning (Skemp, 1989). In other words, learning mathematics is not just a memorization of a collection of rules and chase of correct answers. Instead, students should be fostered their critical thinking skills and fully use of their cognitive operations and thinking operations in conceptualization.

Finally, it is expected that during the process of teaching mathematics, teachers should encourage more teacher/student interaction in class to enhance students' thinking and communication skills (Curriculum Development Council, 2002, p.iv). Teacher questioning in achieving such goal is a matter of critical importance. Hence, teachers are highly recommended to make use of the Qu:Est to nourish student learning in mathematics.

References

Beyer,B.K. (1997). *Improving student thinking: A comprehensive approach*. Boston: Allyn and Bacon.

Bulgar,S., Schorr,R.Y. & Maher,C.A. (2002). Teachers' questions and their role in helping students build an understanding of division of fractions. In Cockburn,A.D. & Nardi,E. (Eds). *International Group for the Psychology of Mathematics Education : PME 26, University of East Anglia, 21-26 July 2002, Norwich UK : proceedings*, 161-168. Norwich : School of Education and Professional Development, University of East Anglia.

Burns,M. (1985). The role of questioning. *The Arithmetic Teacher*, 32(6), 14-16.

Curriculum Development Council (2000). *Mathematics Education Key Learning Area: Mathematics Curriculum Guide (P.1- P.6)*. Hong Kong: Government.

Curriculum Development Council (2001). *Learning to learn: Life-long learning and whole-person development*, p.79. Hong Kong: Government.

Curriculum Development Council (2002). *Mathematics Education: Key Learning Area Curriculum Guide (P.1 – S3)*,p.iii. Hong Kong: Government.

Dantonio,M. & Beisenherz,P.C. (2001). *Learning to question, questioning to learn: Developing effective teacher questioning practices*. Boston: Allyn and Bacon.

Dillon,J.T. (1982). The effect of questions in education and other enterprise. *Journal of Curriculum Studies*, 14(2), 127-152.

Hunkins,F.P. (1976). *Involving students in questioning*. Boston: Allyn and Bacon.

Martino,A.M. & Maher,C.A. (1999). Teacher questioning to promote justification and generalization in Mathematics: What research practice has taught us. *The Journal of Mathematics Behavior*, 18(1), 53-78.

Newton,D.P. (2000). *Teaching for understanding: What it is and how to do it*. London ; New York : Routledge/Falmer.

Ryan,F.L. (1971). Exemplars for the new social studies: Instructing in the elementary school. Englewood Cliffs, New Jersey: Prentice-Hall, Inc.

Skemp,R.R. (1989*). Mathematics in the primary school*. London: Routledge.

Sund,R.B. & Carin,A. (1978). *Creative questioning and sensitive listening technique: A self concept approach*. Columbus, Ohio: Charles E. Merrill.

Tang, K.M. (2002). An investigation on alternative thinking in mathematics education [in Chinese]. *EduMath*, 14, p.28-34.

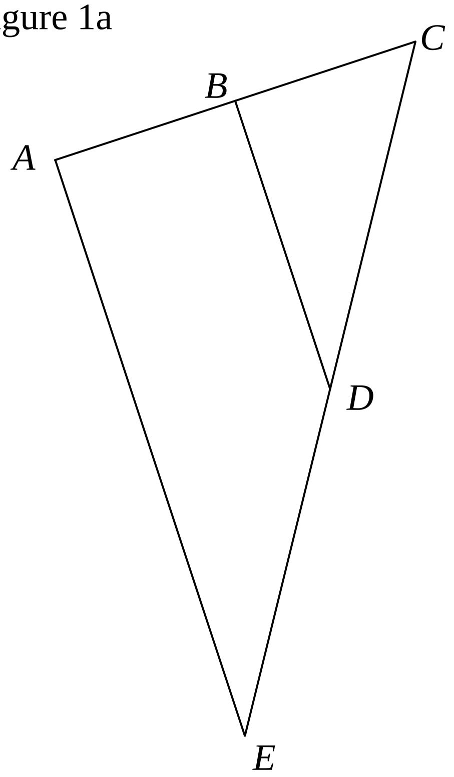
Vace,N.N. (1993). Implementing the professional standards for teaching mathematics: Questioning in the mathematics classroom. *Arithmetic Teacher*, 41(2), 88-91.

Wilen,W.W. (1992). *Questions, questioning techniques and effective teaching* (3rd Ed.). Washington, D.C.: NEA Professional Library, National Education Association.

Wong,N.Y., Lam,C.C., Leung,K.S., Mok,A.C. & Wong,K.M. (1999). *An analysis of the views of various sectors on the mathematics curriculum (Final report)*, p.17-18. Hong Kong: Government.

Appendix I

Figure 1a

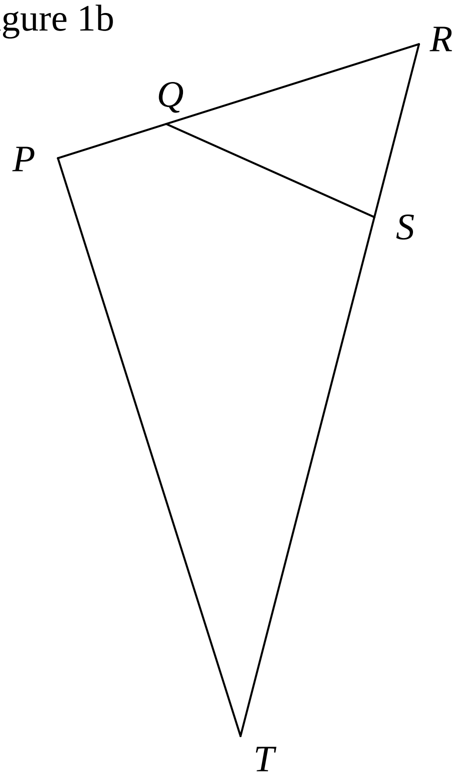


$AB = 2.5$ cm
$BC = 2.5$ cm
$CD = 4.7$ cm
$DE = 4.7$ cm
$BD = 4.0$ cm
$AE = 8.0$ cm

$\angle A = 90°$
$\angle CBD = 90°$
$\angle C = 58°$
$\angle BDC = 32°$
$\angle E = 32°$

Findings:

Figure 1b

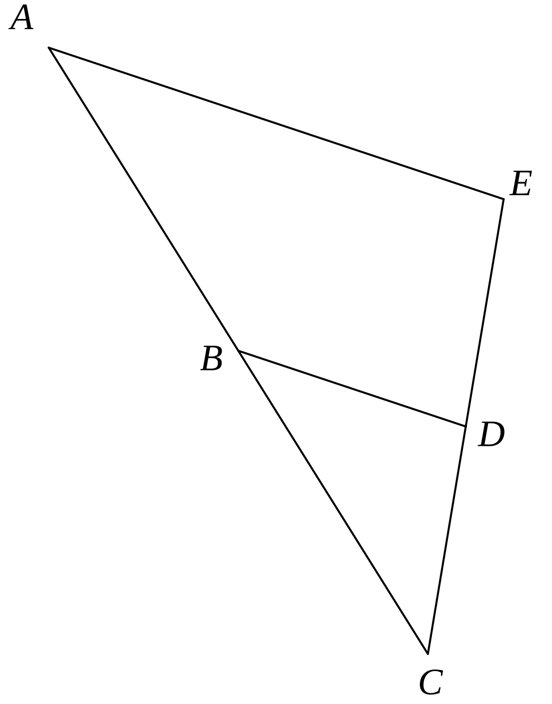


$PQ = 1.5$ cm
$QR = 3.5$ cm
$RS = 2.3$ cm
$ST = 7.1$ cm
$QS = 3.0$ cm
$PT = 8.0$ cm

$\angle P = 90°$
$\angle RQS = 42°$
$\angle R = 58°$
$\angle RSQ = 80°$
$\angle T = 32°$

Findings:

Figure 2a

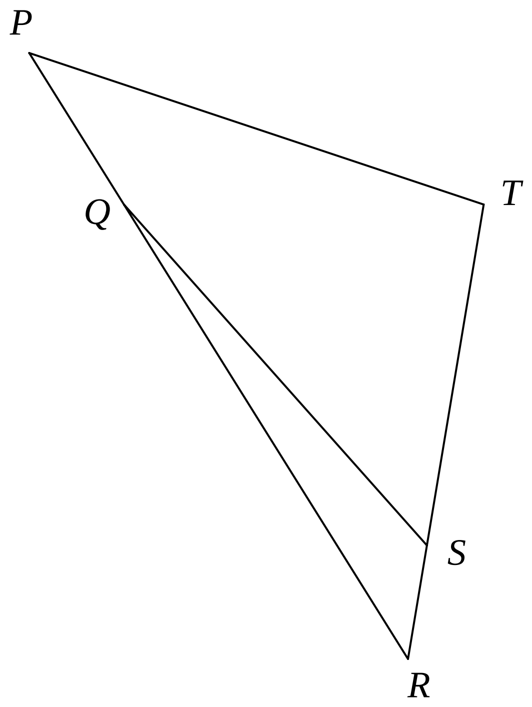


$AB = 4.7$ cm
$BC = 4.7$ cm
$CD = 3.1$ cm
$DE = 3.1$ cm
$BD = 3.2$ cm
$AE = 6.4$ cm

$\angle A = 39°$
$\angle CBD = 39°$
$\angle C = 42°$
$\angle BDC = 99°$
$\angle E = 99°$

Findings:

Figure 2b

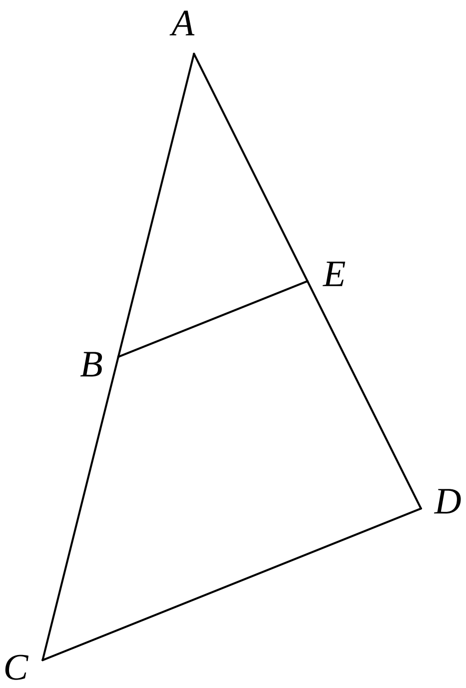


$PQ = 2.3$ cm
$QR = 7.1$ cm
$RS = 1.6$ cm
$ST = 4.6$ cm
$QS = 6.0$ cm
$PT = 6.4$ cm

$\angle P = 39°$
$\angle RQS = 11°$
$\angle R = 42°$
$\angle RSQ = 130°$
$\angle T = 99°$

Findings:

Figure 3a

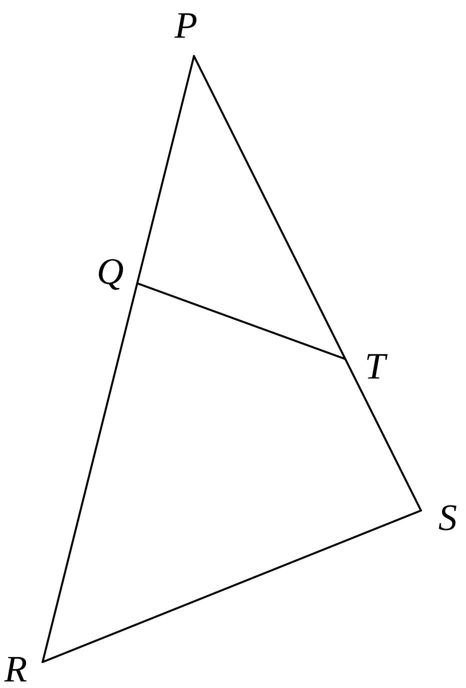


$AB = 4.1$ cm
$BC = 4.1$ cm
$AE = 3.4$ cm
$ED = 3.4$ cm
$BE = 2.7$ cm
$CD = 5.4$ cm

$\angle A = 41^\circ$
$\angle ABE = 54^\circ$
$\angle C = 54^\circ$
$\angle AEB = 85^\circ$
$\angle D = 85^\circ$

Findings:

Figure 3b



$PQ = 3.1$ cm
$QR = 5.1$ cm
$PT = 4.5$ cm
$TS = 2.3$ cm
$QT = 2.9$ cm
$RS = 5.4$ cm

$\angle P = 41^\circ$
$\angle PQT = 95^\circ$
$\angle R = 54^\circ$
$\angle PTQ = 44^\circ$
$\angle S = 85^\circ$

Findings:

Appendix II