Three solutions (without words) to a single question

Tsui Wai Kwok Ricky (PLK Ma Kam Ming College)

Chan Chui See Tracey (St. Paul's Co-educational College)
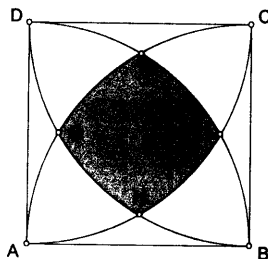
Lee Man Sang Arthur (St. Joseph's College)

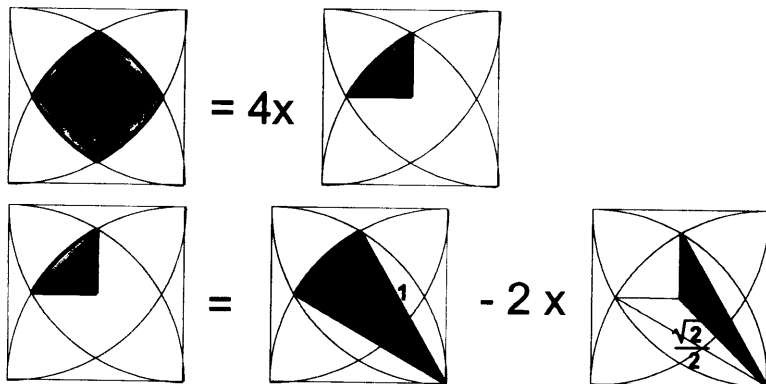Li Siu Kei Ricky (B.Ed. Math student, CUHK)

Problem

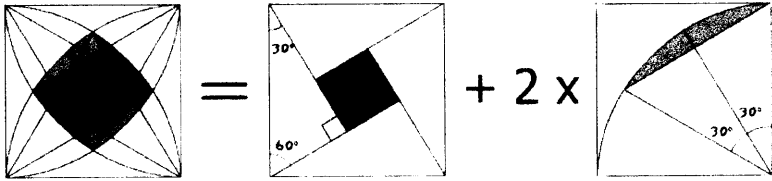*ABCD* is a unit square. *DFIB*, *CFGI*, *DGHB* and *CIHA* are arcs of unit circle. Find the area of the shaded part.
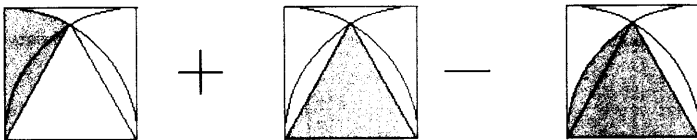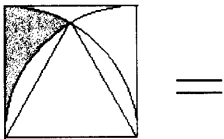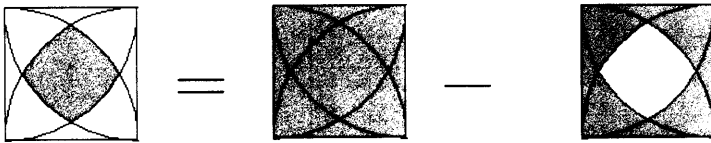

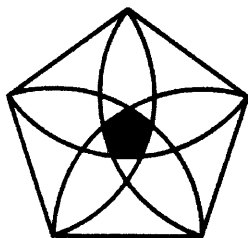**Solution I (Tsui Wai Kwok Ricky and Chan Chui See Tracey)**

Solution II (Lee Man Sang Arthur)

Solution III (Li Siu Kei Ricky)

Variations

What is the perimeter of the shaded portion?

(taken from Japanese Primary Math Olympia: 雅孝司(2001)(編)。《數學奧林匹克》。台北：益智工房。)

Other variations include:

1. Use other regular polygons;

2. Use different lengths as radius, e.g. One of the diagonals;

3. Instead of vertices, use mid-points of the sides as centres.

Editorial Note:

Yip So Ling discussed a series of 4 problems in the classroom context in Datum, 1995, in which the fourth one is the same as the one we pose here.

Interested readers are referred to "葉素玲(1996)。課堂小記。Datum, 35, 18-25"for details.