Graphical Calculators in Mathematics Education

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Graphical calculators have a significant impact on the teaching and learning of mathematics at secondary and college level. They will bring about some changes in the way we teach the subject and also some changes in the subject content.

The microcomputer has not had the impact that many people expect on the teaching and learning of mathematics. The reasons are several. One of them is that microcomputers are not yet widely available in schools due to their cost. Another reason is that software packages in mathematics are still limited in supply and they are also very expensive. Furthermore, teachers are not very sure of the role of the microcomputers in the classroom.

Graphical calculators in mathematics as compared to microcomputers have the advantages of being easily portable, relatively cheap, quite user-friendly and powerful. They can do numerical and simple symbolic manipulations as well as function graphing. They are dedicated to mathematics because they do not do other things such as word processing so that mathematics teachers in schools need not share their use with language teachers thus avoiding any dispute over computer usage.

However, while their graphing capabilities are reasonably satisfactory, their existing symbolic capabilities are rather restricted due to their small memory and slower processing speed as compared to microcomputer algebra systems such as DERIVE, MAPLE and MATHEMATICA. In spite of the shortcomings, graphical calculators are effective graphing tools. They can graph functions easily and quickly and enable the user to use graphs to solve equations. They are also good learning devices because they make it possible for students to tackle more graphing problems and allow students to concentrate on conceptual understanding rather than on techniques or skills. They promote a more experimental approach to learning mathematics by allowing students to explore using their graphical capabilities.

According to the NCTM Curriculum and Evaluation Standards for School Mathematics, calculators should be made available to all students at all times. The availability of graphical calculators raises an

important question : What mathematics should be taught and how should it be taught? In other words, do students still need to learn the same pencil-and-paper manipulative algebra such as equation solving and curve sketching taught in the same way as before? The answer to the latter question is obviously no but it does not mean that the topics will be deleted from the school mathematics curriculum or less manipulative algebra will be taught.

On the contrary, teachers have to teach more but the emphasis will not be the same. Students also have to learn more. They have to, first of all, learn to use graphical calculators and learn how these machines can be used to draw graphs of functions and to solve equations. In graph plotting using graphical calculators, students must be aware of the importance of choosing an appropriate viewing window and should know how to interpret the graphs drawn critically. To do that, students need to acquire a better understanding of the mathematical concepts involved.

There is a misunderstanding among some teachers that with graphical calculators there is no need to teach curve sketching and solving equations by algebraic and numerical methods. In fact, the sketching of graphs will remain important. A graph sketch should not be taken as a rough plot. It is a manifestation of mathematical knowledge such as asymptotes, discontinuity, intercepts, symmetry, turning points and so on. A sketch of a graph is also useful in checking whether the graph drawn by a graphical calculator is correct or not. To use graphical calculators effectively students must be able to judge the reasonableness of results produced by the machines. This is similar to the case of ordinary calculators in which an ability to estimate computations is essential. Curve sketching and calculator graphing are supplementary to each other.

In equation solving there is still a need to teach how to solve simple equations using algebraic methods and to solve certain non-linear equations using simple numerical methods. Learning these methods enables students to solve some simple equations and use the methods to compare the results obtained from graphical calculators. For more complicated equations, we can leave them for graphical calculators to solve. It is so easy for graphical calculators to graph a function, to zoom in for greater details, to zoom out to view the function for its global characteristics and to obtain the zeros of the function using the trace function of the calculator. Such a graphical approach is not to replace algebraic approach but to be used to foster better understanding of the latter.

However, with the availability of graphical calculators, the emphasis on equation solving, function graphing and manipulative algebra will no longer be the same. Students will still be required to demonstrate some basic ability in algebraic manipulations, solving simple equations and sketching some curves as demanded by public examinations and standardized tests. The use of graphical calculators permits students to solve more equations and more complex ones, to graph more functions and to explore the behaviour of many classes of functions. The focus will not be on mastering manipulative and graphing skills in algebra, since these can be more quickly and accurately done with graphical calculators. The emphasis will be on the conceptual understanding of functions and graphs, the setting up of equations from problems, the meaning of roots of equations so that students can verify by substitution whether the solutions of equations obtained from graphical calculators are correct or not and how to get information from graphs.

Less emphasis will be placed on graph plotting. There is no need to drill students in drawing graphs by computing ordered pairs and then plotting the points as this is a very time consuming task but there is a need for them to do a few examples of simple graph plotting in order that they can understand the connection between a function and its graph, the properties of the graph and its use in problem solving.

Several graphical calculators are now available in the market. The popular ones are produced by CASIO, SHARP, Hewlett-Packard and Texas Instruments. Later models (TI-85, CASIO FX7700GB, SHARP EL9300C and HP 48G) are increasingly powerful, more expensive and more difficult to master.

In order to help our students use graphical calculators effectively, teachers need to learn how to use them, analyse what they can do, what topics of mathematics would benefit most form using the technology and determine how the machines might be used to teach the topics effectively. All mathematics teacher education programmes should include a component on using graphical calculators and computer algebra systems in mathematics.

I believe that the graphical calculator is only a tool in mathematics. It should be used to enhance but not to replace the teaching of mathematics. It should be used for only what it is good at. Lessons involving no technology but have been shown to be more successful need not be recast for the sake of employing the technology. At present, graphical calculators are good at graphing functions and root-finding.

They are useful in providing students the opportunity to observe patterns in graphs and to explore the effect of changes of a, h and k in

$$y = a\, f(x + h) + k$$

on the function f(x) such as $f(x) = x^2$, sin x etc. The ease and speed with which the graphs can be drawn with graphical calculators make the exploration appealing to students. As the power of graphical calculators continues to increase, more and more capabilities will be added. Future models will be able to manipulate symbolic expressions including partial fractions, differentiation, integration, solving differential equations and so on. When this is realized, the learning of many more topics in the school mathematics curriculum will be enhanced and made more enjoyable with the technology. Perhaps, such new graphical calculators enhanced with capabilities of a computer algebra system should be called supercalculators.

At this point, I must say that I am for the use of the technology in the classroom. However, there are still two main obstacles to the use of graphical calculators in the classroom before the technology can be integrated with the mathematics curriculum. The first obstacle is cost. Although graphical calculators are relatively cheap compared to microcomputers, they are still expensive. The price of graphical calculators must come down to the level at which most if not all students can afford to own the machines. They must be reasonably powerful but easy to use, more user-friendly and having simpler keyboard than existing models. Ideally, graphical calculators should be available under several categories, something like the ordinary calculators which are classified into three levels: simple four-function calculators for elementary school pupils, scientific calculators for secondary and programmable calculators for high level students. The second obstacle is public examinations and standardized tests which largely determine the content and emphases of school mathematics. The examination syllabuses and test items which influence textbooks and other teaching materials need to be modified to incorporate the use of graphical calculators.

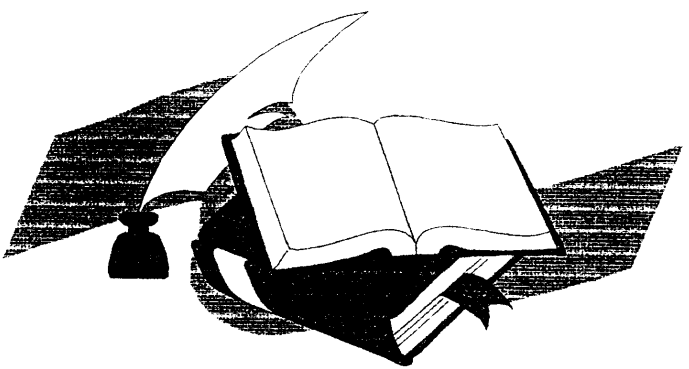
If we look back in the history of calculators, the pocket electronic calculator first came to the market in Singapore, Malaysia and Hong Kong in the early 1970s and not until about ten years later, the calculator was finally accepted into the secondary school mathematics curriculum and this happened only when the cost of a calculator dropped considerably and the curriculum was revised with the permission of calculator use in examinations. The future of graphical calculators is bright but it will take at least five years form now before we can see

widespread use of the technology in the classroom as ordinary scientific calculators are being used today *provided that the two obstacles are removed.*

In the meantime, we should encourage teachers at all levels to experiment with graphical calculators either in their own research to find out what areas of mathematics these tools are best for or in some classroom activities that exploit the use of the technology to enhance the teaching and learning of mathematics.

Reference

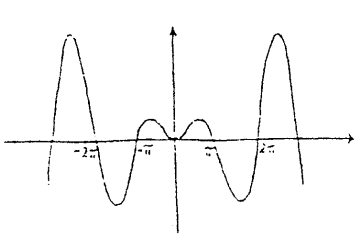
National Council of Teachers of Mathematics (1989). ***Curriculum and Evaluation Standards for School Mathematics*** , Reston,VA : NCTM.

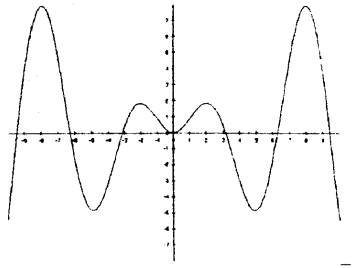


Appendix

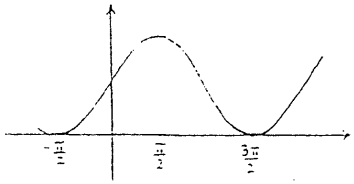
Some hand sketches and graphs drawn by a graphical calculator are shown below:

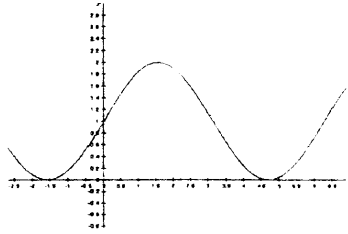
$y = x \sin x$



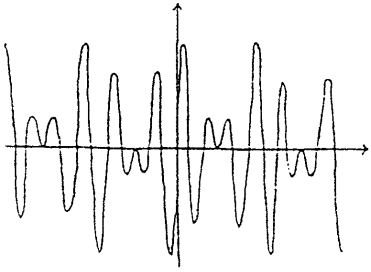


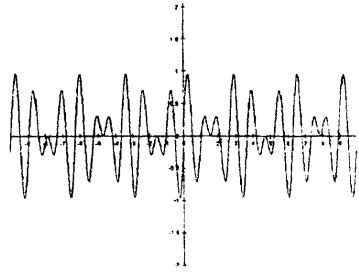
$y = 1 + \sin x$



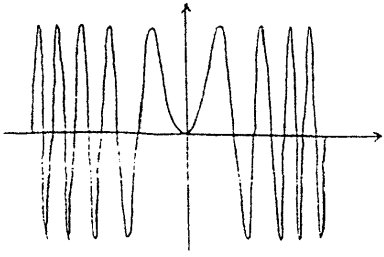


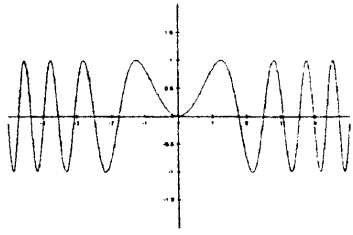
$y = \sin 6x \cos x$



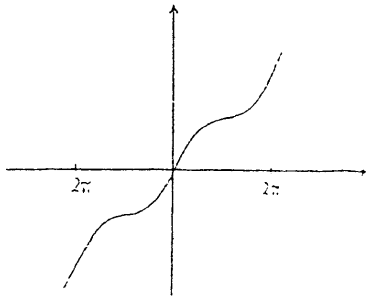


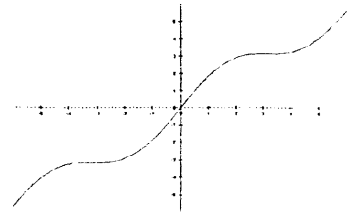
$y = \sin (x^2)$



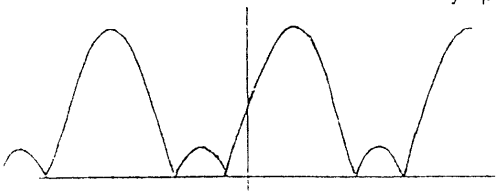


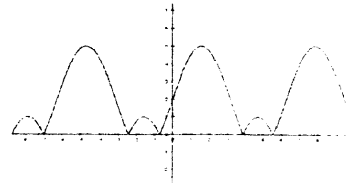
$y = x + \sin x$



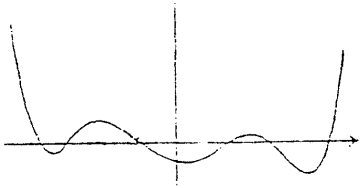


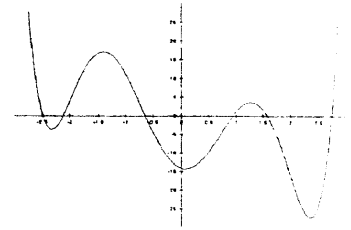
$y = |3 \sin x + 2|$



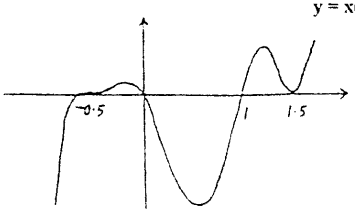


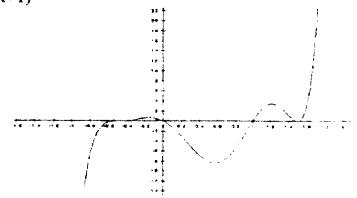
$y = x^6 - 11x^4 + 30x^2 - 5x - 14$





$y = x(x-1)(2x-3)^2(2x+1)^3$





$$y = \frac{x^2 - 1}{x^2 + 1}$$