A case study on fostering mathematical thinking by dynamic geometry exploration

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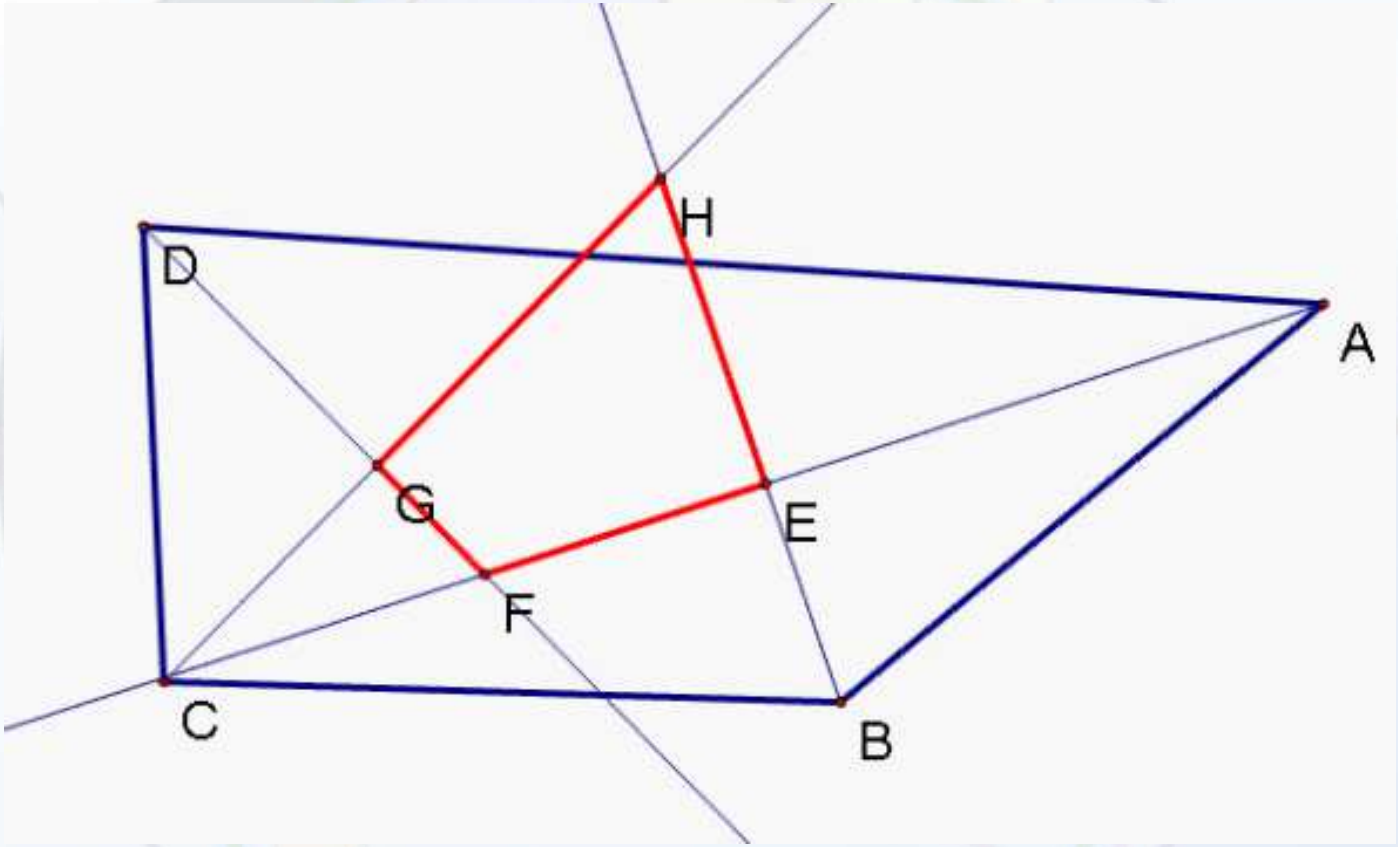
The task

- *Consider a quadrilateral ABCD. Construct the angle bisectors of each angle of the quadrilateral. The intersections of these angle bisectors form a new quadrilateral EFGH. Use Sketchpad to investigate the properties of quadrilateral EFGH and its relationship with quadrilateral ABCD.*
- *You may use any exploration techniques and dragging strategies learnt in earlier sessions. Make as many conjectures as possible and write down what you have found.*

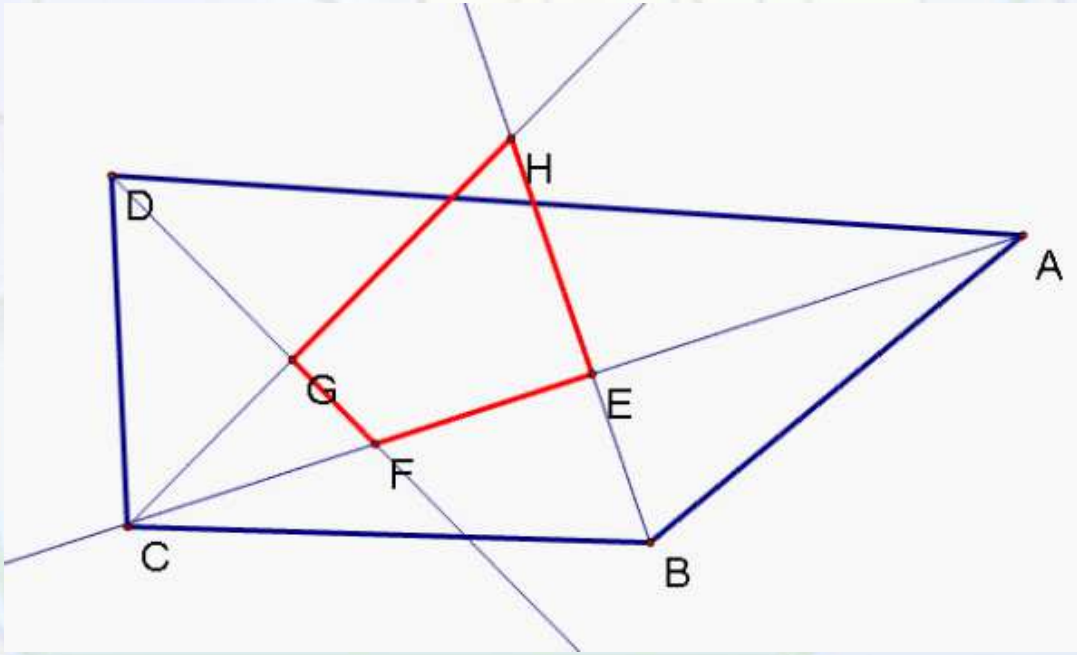
The task



- Remark: EFGH is called bisectogram of ABCD.

Wait !

- *How will you do this task?*
- *Can you guess what conjectures can be made?*



Context

- The is a case study taken from data collected in my PhD research project finished in 2009.
- The overall aim of the research project was to investigating the problem solving strategies in dynamic geometry software (DGS).

Chan, Y.C. (2009). *Experimental-theoretical Interplay in Dynamic Geometry Environment*, Unpublished PhD dissertation, Hong Kong: University of Hong Kong.

Context

- The participants met the researcher individually once a month for a period of 1 year:
- Geometric explorative task + post-task interview
- This was the 8th task.

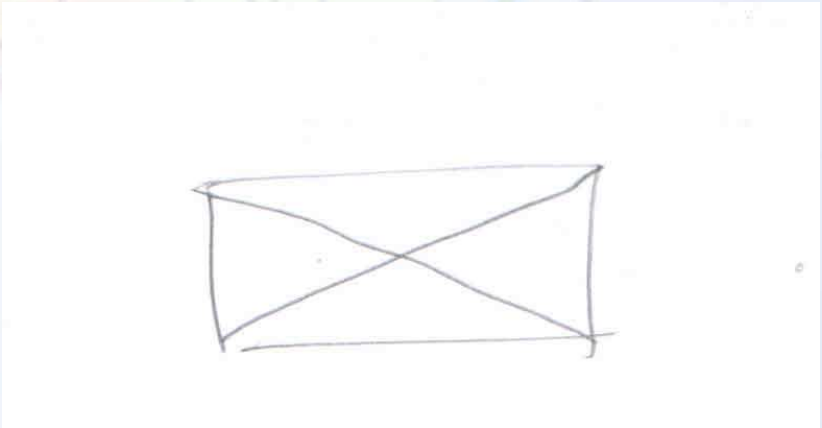
Background information of the participant reported in this presentation

- Candy (pseudonym)
- A female final year undergraduate student majoring in mathematics (with computer science option)

Candy's thinking process

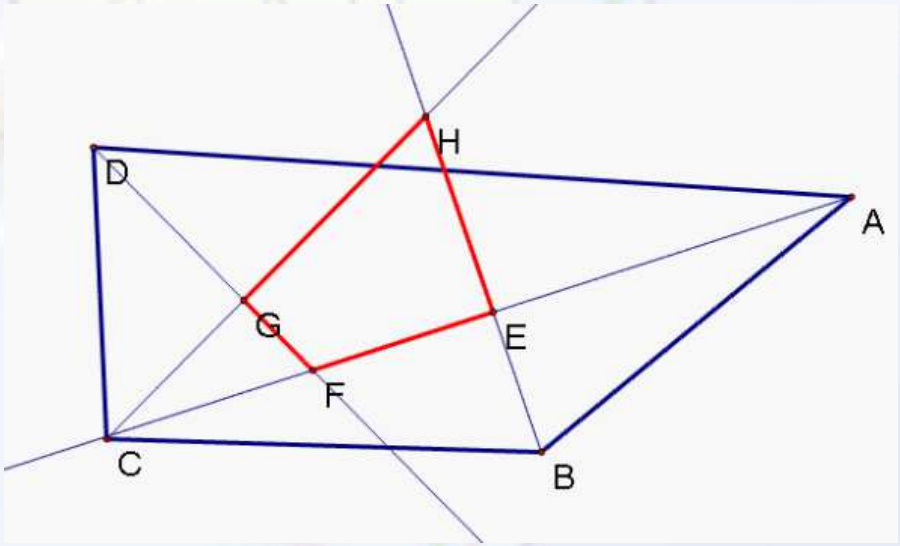
1. Incorrect knowledge led to exploration of the case of an arbitrary quadrilateral

- Mistakenly thought that the angle bisectors of the angles of those special quadrilaterals (e.g. squares, rectangles, parallelogram) are the diagonals and hence bisectograms cannot be formed.



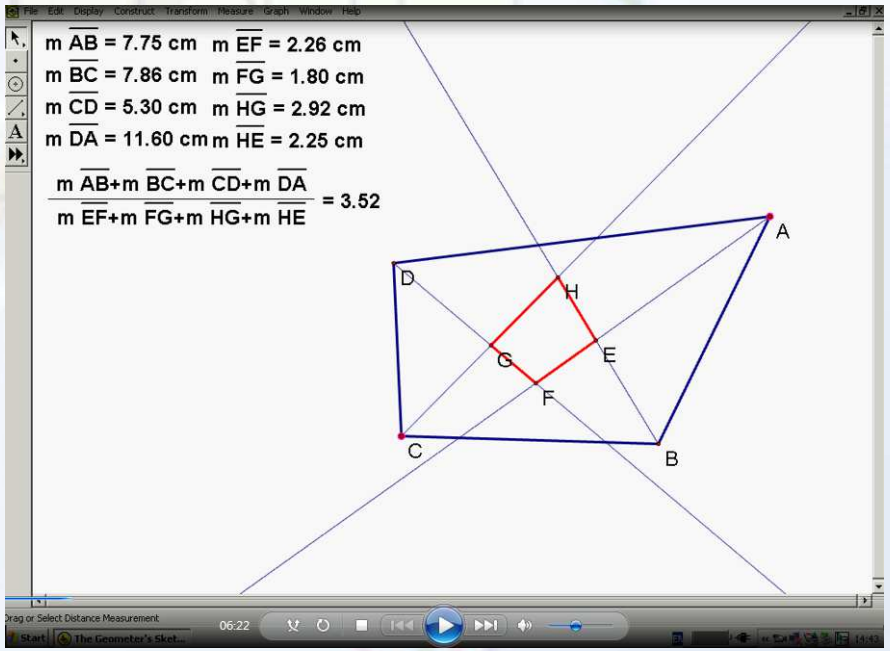
1. Incorrect knowledge led to exploration of the case of an arbitrary quadrilateral (con't)

- Explore the case of an arbitrary quadrilateral, instead of special cases.



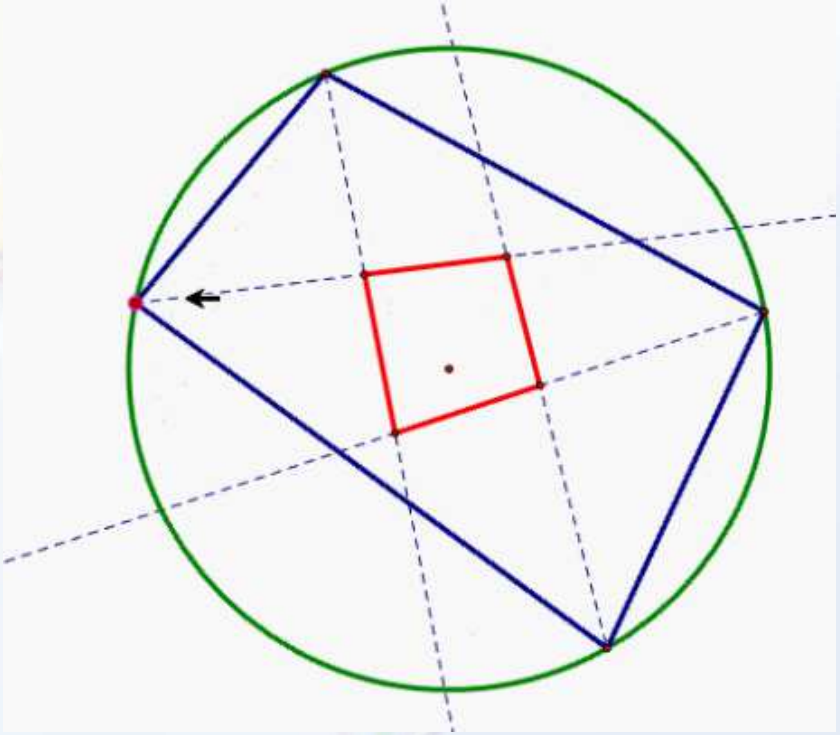
2. Attempted to find the relationships of the lengths of the sides and of the sizes of the angles

- Measuring tools:
- Playing around these lengths of sides and the sizes of the angles of the two quadrilaterals by doing some kind of ‘random computations’.



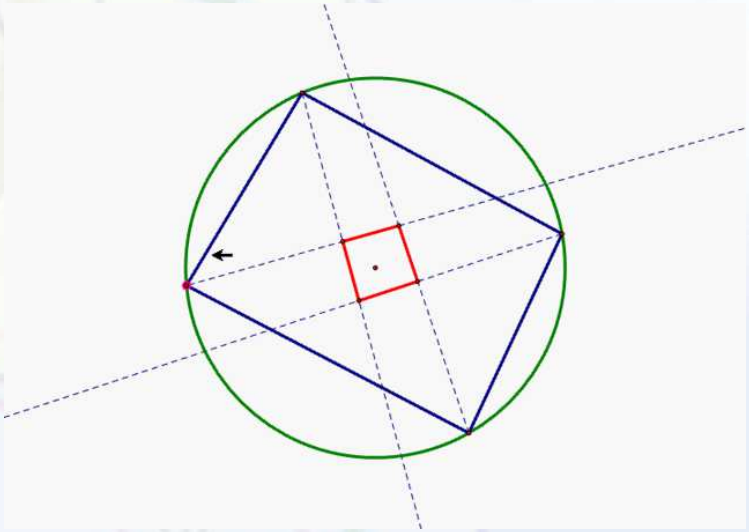
3. Considered the case of a cyclic quadrilateral

- Dragged the vertices of quadrilateral ABCD and ‘played around’ with the lengths of the sides and the sizes of the angles of the two quadrilaterals.



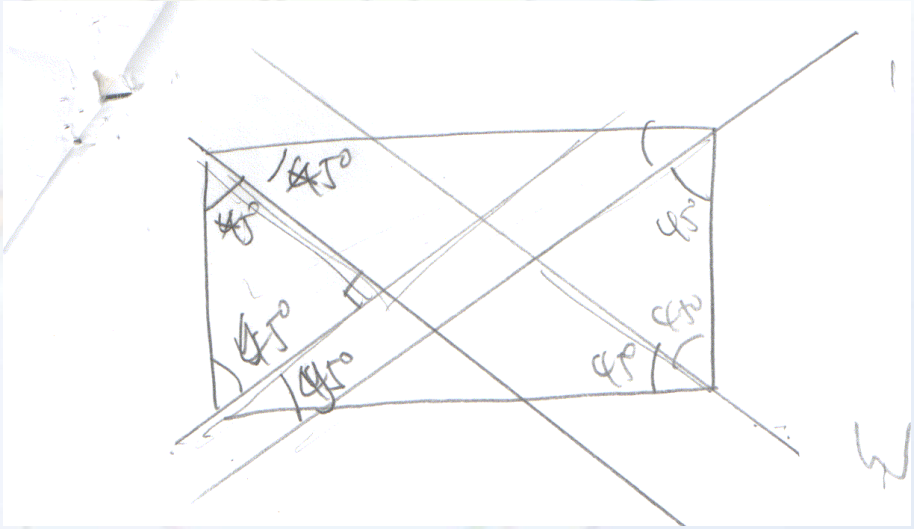
3. Considered the case of a cyclic quadrilateral (con't)

- As the vertices were bounded along the circle as so constructed, dragging became systematic.
- After dragging for a while, quadrilateral ABCD became a rectangle. She realized that the angle bisectors did mot meet at a point and hence a bisectogram can be formed.



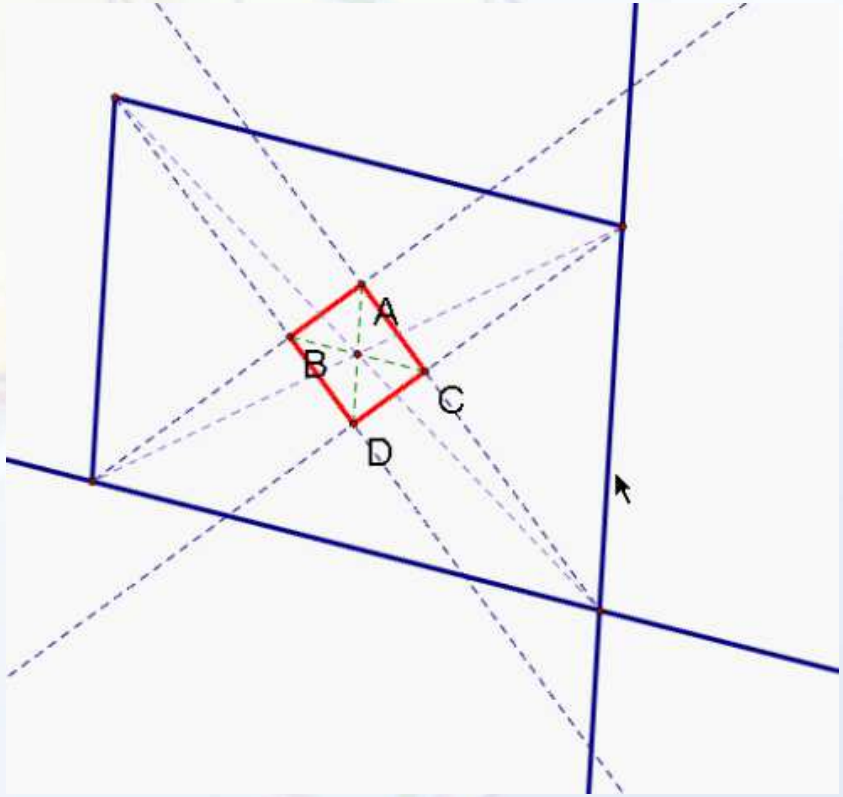
4. Considered the case of a rectangle

- Drew the configuration on the task sheet and did some simple computations
- Pushed up to a theoretical level rather than just a somewhat trial-and-error random exploration

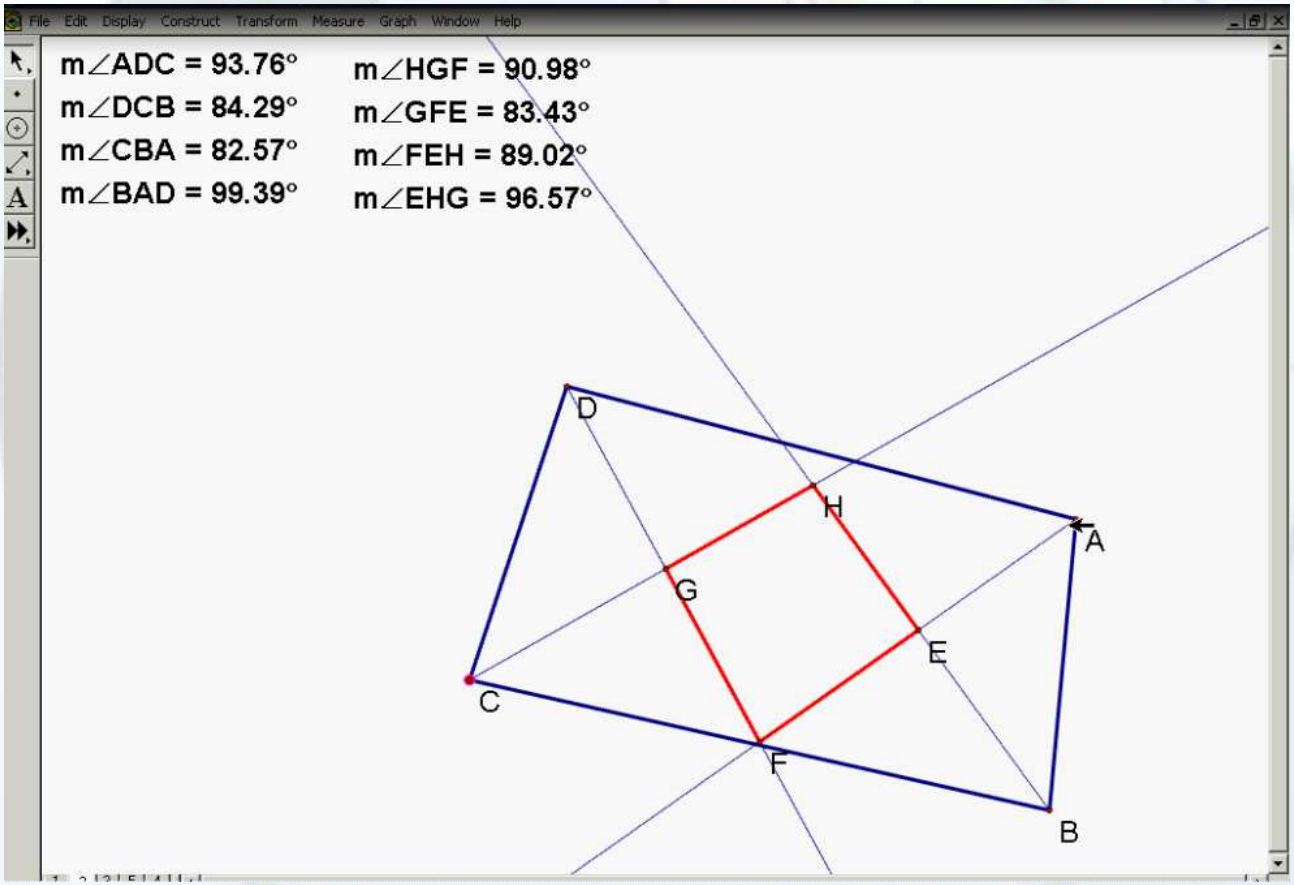


5. Considered the case of a parallelogram

- Observed that the four diagonals meet at a point and the diagonals of the rectangle are parallel to (one pair of) the sides of the parallelogram



6. Discovered that the quadrilateral formed by the angle bisectors of a quadrilateral must be cyclic



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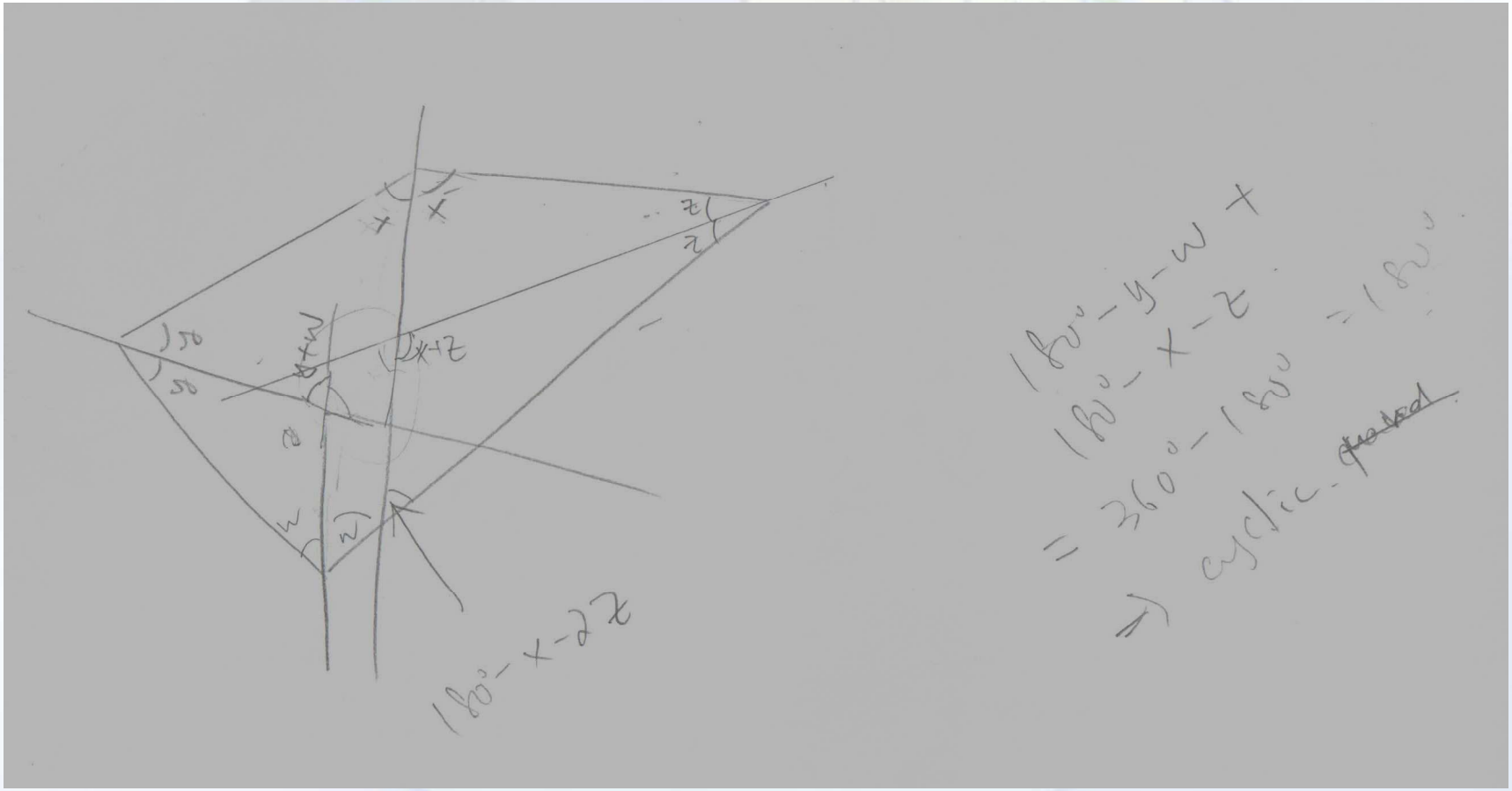
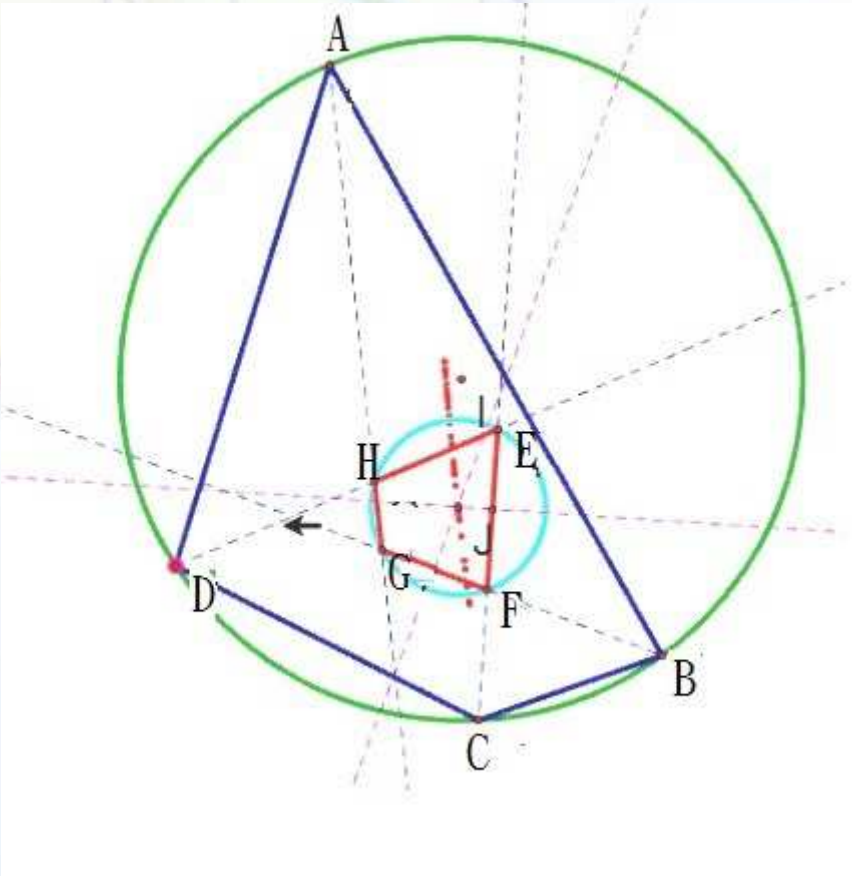


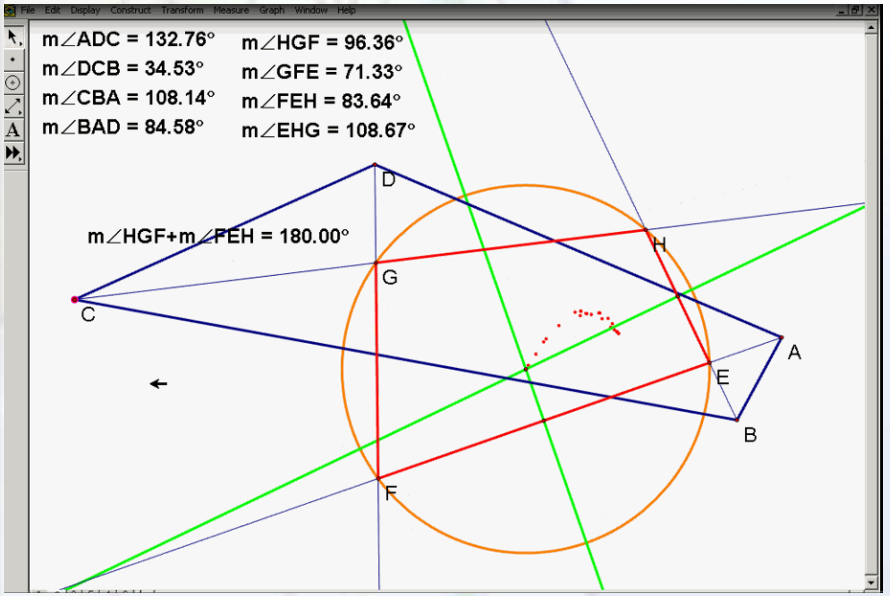
7. Further investigated the cyclic quadrilateral formed by the angle bisectors

- Used *trace* to display the locus of the center of the circumscribed circle of bisectogram EFGH while dragging the vertices of quadrilateral ABCD
- Found that the locus of the center is a straight line when quadrilateral ABCD is cyclic



7. Further investigated the cyclic quadrilateral formed by the angle bisectors

- This property does not hold for an arbitrary quadrilateral (i.e. when quadrilateral ABCD is not cyclic).



Properties found in this task

Property 1

- Suppose quadrilateral ABCD is a parallelogram. Then, the quadrilateral EFGH formed by its angle bisectors is a rectangle. Furthermore, the diagonals of parallelogram ABCD and the diagonals of rectangle EFGH meet at a point. The diagonals of the rectangle EFGH is parallel to (one pairs of) the sides of the parallelogram ABCD.

Property 2

- The quadrilateral EFGH formed by the angle bisectors of the four angles of (any type of) a quadrilateral ABCD must be cyclic.

Property 3

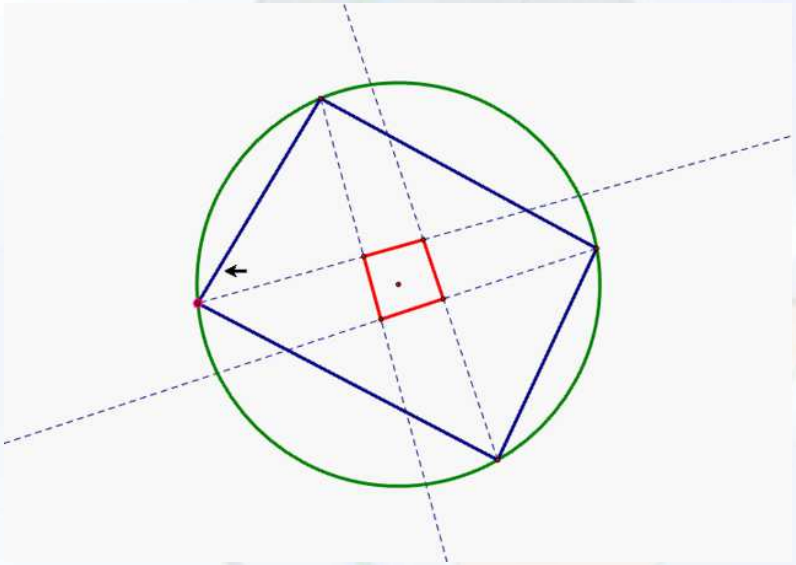
- If one of the angles of ABCD is greater than 180 degrees (in this case, she called quadrilateral ABCD ‘reflexive’), then EFGH is not a quadrilateral.

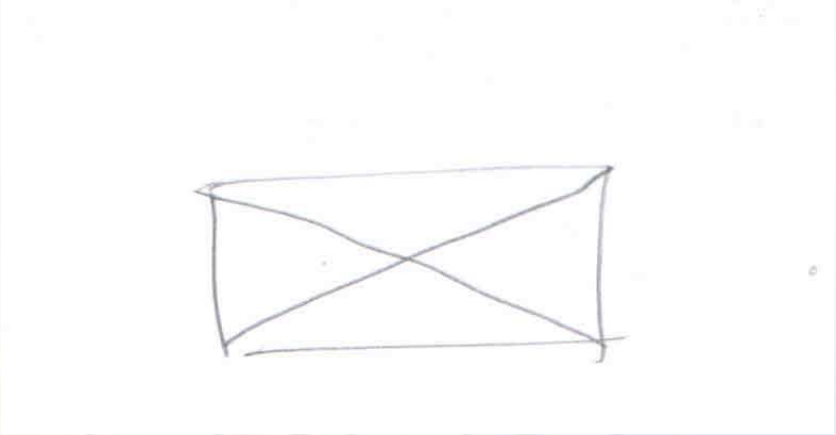
Property 4

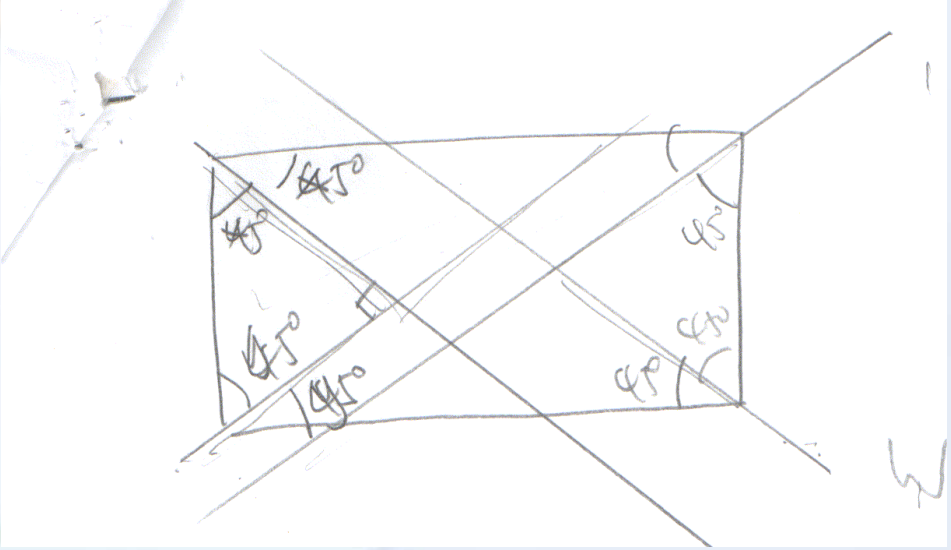
- Suppose quadrilateral ABCD is a cyclic quadrilateral. When one of the vertices of quadrilateral ABCD moves along its circumscribed circle and keeping the other vertices fixed, then (the locus of) the center of the circumscribed circle of quadrilateral EFGH is a straight line.

Critical moments

Re-evaluated her prior knowledge







Re-evaluated her prior knowledge

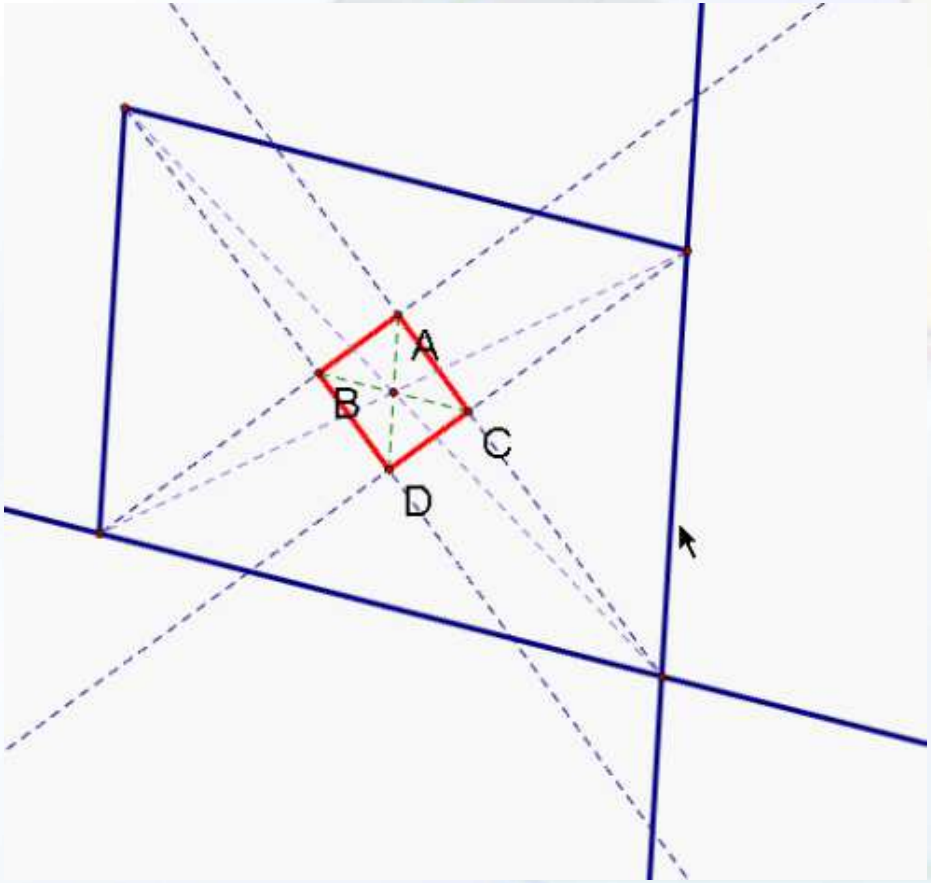
下午 做 Research，才發現...

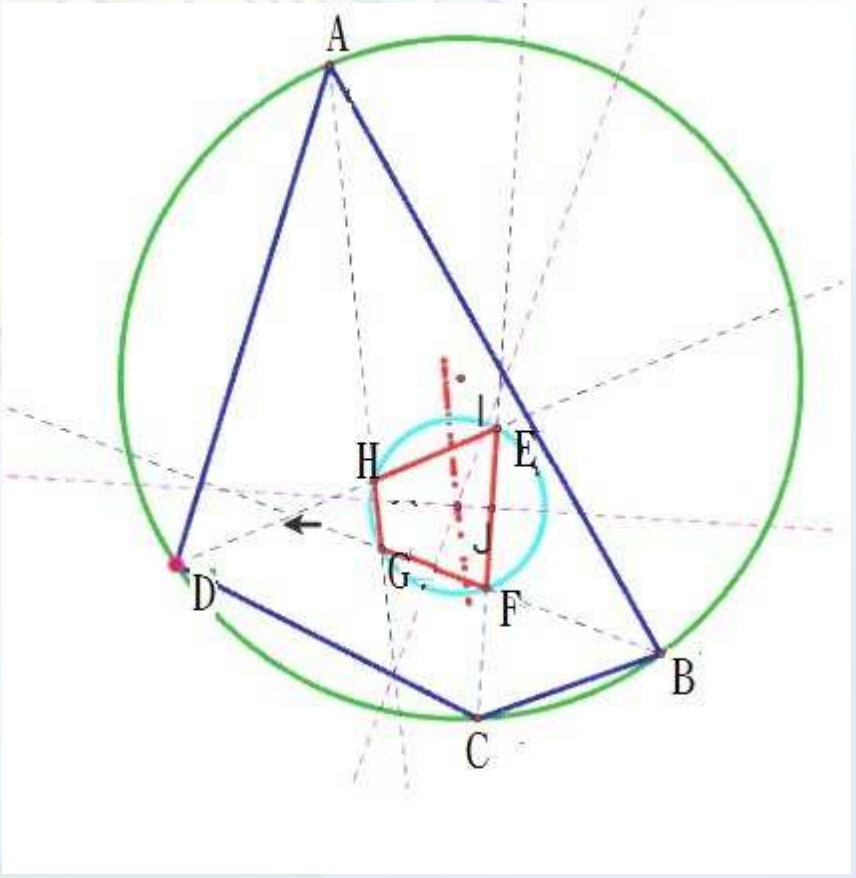
原來長方形和正方形是圓內接四邊形（cyclic quad.）來啊！

還有！！！長方形和平行四邊形的對角線原來不是平分每隻角！！

這件事，千萬不可以給學生知道...

Made an analogue





What have we learnt from this story?

Hints on designing good DGS exploration tasks for fostering mathematical thinking

- *Well-designed DGS exploration task can remind mathematical knowledge which is usually overlooked.*
- *Well-designed DGS exploration task should make use of the dynamic feature of the software. For instance, trace command should be encouraged to use.*

Hints on designing good DGS exploration tasks for fostering mathematical thinking (con't)

- *Using vague vocabularies in the question stem of a task may not be a bad thing. Creativity may be encouraged which may in turn lead to discovering of unexpected mathematical properties.*
- *Using DGS to do exploration does not rule out the need of using other 'traditional' tools such as paper-and-pencil. Paper-and-pencil is still an important tool for theoretical thinking such as seeking for a mathematical proof.*

Acknowledgement

- The empirical study reported in this paper is part of the author's PhD research study conducted at the University of Hong Kong under the supervision of Dr Allen Leung.

Thank you!

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