電子科技與
體現數學學習

科技變革還是教學法變革？(1 of 3)

April 21, 2021

About Me

My first year of teaching [2005]

Teaching with various tools [2005-2012]

AP Calculus teaching [2012]

PhD studying [2012-2016]

Researching at CUHK [2012-2016]

Plenary Speaker at PME Conference [2021]

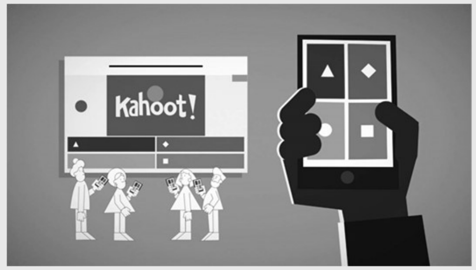
iPad [2010]

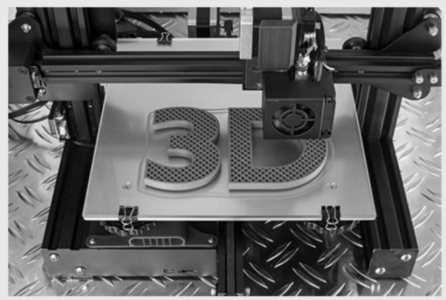
iPad 3 [2012]

iPad 3 [2013]

iPad 7 [2015]

iPad 9 [2017]



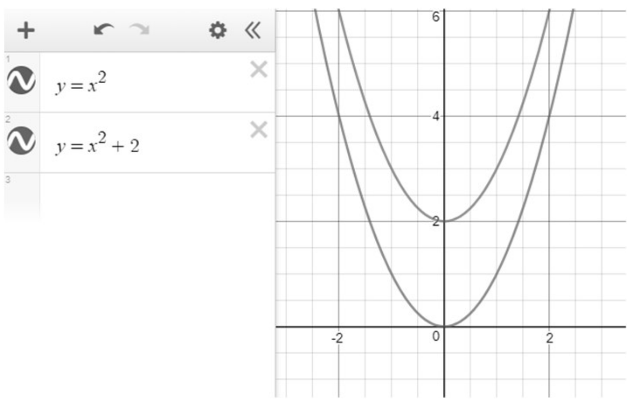


What is “transformation” (變革)

Q: How transformative is…
[rate from 1-5]

What is “transformation” (變革)

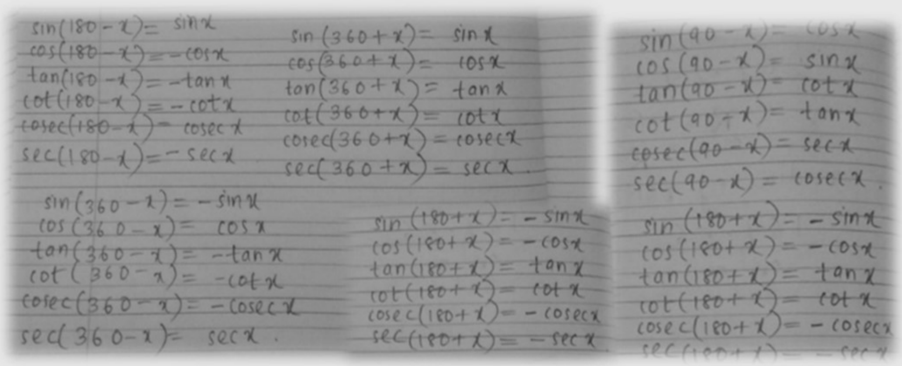
- **What it is… (for me)**



  - $y = f(x) \rightarrow y = a\,f(bx+c) + d$ [property change]
  - 將數學教育從線性、抽象和紙筆為主的特徵轉變為包含多種模式、動手實踐和應用現代科技的特徵。
  - Transforms what and how mathematics is learned
  - E.g.: trigonometric identities with/out technology? [https://www.geogebra.org/m/asrzp73d#material/beturhmk]

- **What it's NOT…**
  - Efficiency
    - Transformation vs. efficiency/productivity
  - (always) better exam scores
    - conceptual vs. procedural understanding

Technology-rich vs. technology-enhanced mathematics learning



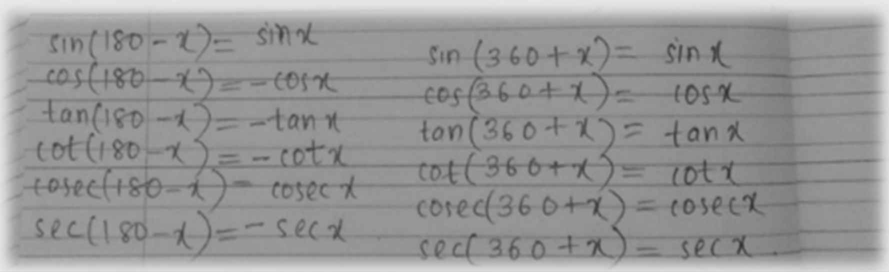
Sinclair (2017)

History of tools in Maths

School mathematics?

Using 1400AD technology in an era where practitioners use 1950s technology; demonising pre-Greek technology and delaying Classical Greek technology!

(Sinclair, 2017)



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5	4	3	2	1
١٠	٩	٨	٧	٦
10	9	8	7	6

Tools for Maths thinking, learning & teaching…

- **Mystery Transformation:**
  - https://www.geogebra.org/m/vB8YGB5q#material/NmGbexAG

- **Thinking and learning → teaching**
  - Thinking: engaging in mathematical ideas
  - Learning: a change in doing/talking mathematics
  - Teaching: Occasion opportunities to think and learn mathematics

- **Fundamental ideas:**
  - What can technology and tools afford that paper-and-pencil cannot?
  - From tool-based mathematics thinking…
  - …to tool-based pedagogy

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(1) Gesturing and Thinking

Educ Stud Math (2009) 70:127–141
DOI 10.1007/s10649-008-9124-6

Gestures and conceptu

Laurie D. Edwards

Educ Psychol Rev (2015) 27:405–412
DOI 10.1007/s10648-015-9325-3

CrossMark

REVIEW ARTICLE

Learning from Gesture: How Our Hands Change Our Minds

Miriam Novack[1] · Susan

ıent in Mathematics Teaching
ing: Evidence From Learners'
ıd Teachers' Gestures

International Journal of Mathematical Education in Science and Technology, Vol. 42, No. 7, 15 October 2011, 891–901

Taylor & Francis
Taylor & Francis Group

Gestures and insight in advanced mathematical thinking

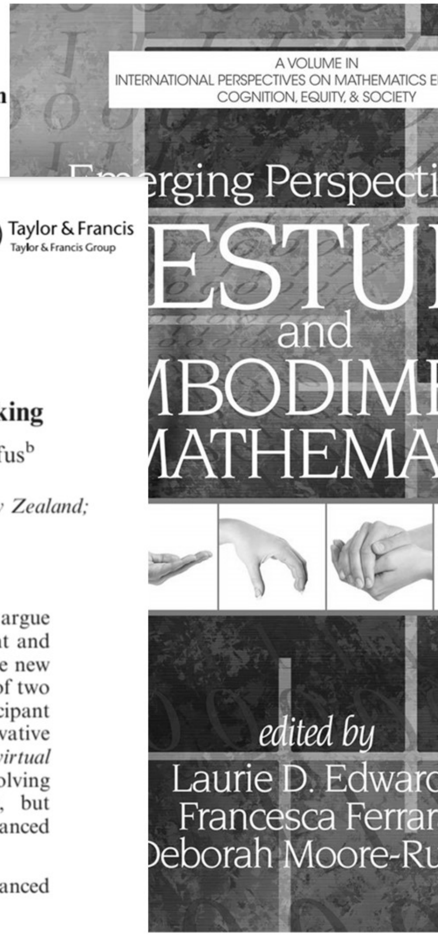
Caroline Yoon[a]*, Michael O.J. Thomas[a] and Tommy Dreyfus[b]

[a]*Department of Mathematics, The University of Auckland, Auckland, New Zealand;* [b]*School of Education, Tel Aviv University, Tel Aviv, Israel*

(*Received 30 May 2011*)

What role do gestures play in advanced mathematical thinking? We argue that the role of gestures goes beyond merely communicating thought and supporting understanding – in some cases, gestures can help generate new mathematical insights. Gestures feature prominently in a case study of two participants working on a sequence of calculus activities. One participant uses gestures to clarify the relationships between a function, its derivative and its antiderivative. We show how these gestures help create a *virtual mathematical construct*, which in turn leads to a new problem-solving strategy. These results suggest that gestures are a productive, but potentially undertapped resource for generating new insights in advanced levels of mathematics.

Keywords: gesture; calculus; virtual mathematical construct; advanced mathematical thinking; mathematical insight



Educ Stud Math (2013) 83:285–307
DOI 10.1007/s10649-012-9454-2

The interplay among gestures, discourse, and diagrams in students' geometrical reasoning

Chia-Ling Chen · Patricio Herbst

Published online: 28 December 2012

Abstract This study explores interactions with diagrams that are involved in geometrical reasoning; more specifically, how students publicly make and justify conjectures through multimodal representations of diagrams. We describe how students interact with diagrams using both gestural and verbal modalities, and examine how such multimodal interactions with diagrams reveal their reasoning. We argue that when limited information is given in a diagram, students make use of gestural and verbal expressions to compensate for those limitations as they engage in making and proving conjectures. The constraints of a diagram, gestures and linguistic systems are semiotic resources that students may use to engage in geometrical reasoning.

Keywords Gesture · Geometrical reasoning · Diagrams · Systemic functional linguistics · Semiotics · Conjecture

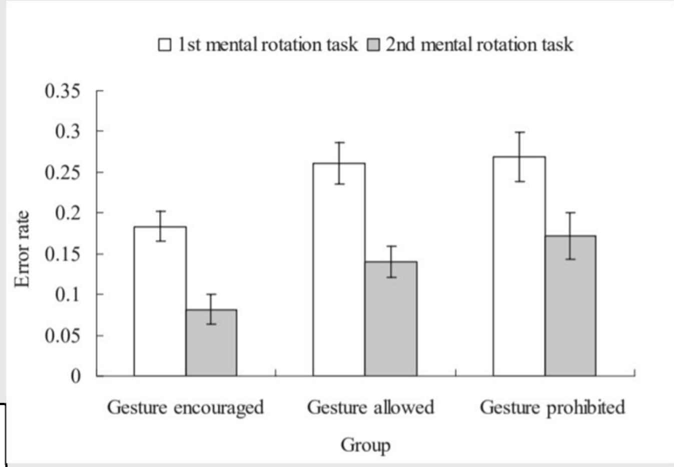
Gesturing and Thinking

Try: Explain the Sine Law to a colleague.

- (by sitting on your hands)

Now try: 345x67

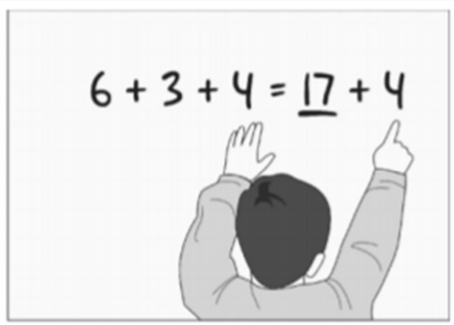
- **Evidence that your hands are thinking:**
  1. Spatial reasoning with gestures (Ehrlich, et al., 2006)
     - Gestures accompany spatial information in speech
  2. Finger Perception (Boaler, 2016)
     - College students' finger perception predicts calculations scores
     - Finger perception in grade 1 is better prediction of math achievement in grade 2 than test scores
  3. Gestures contribute to effective communication (Alibali & Nathan, 2012)
     - Students learn better with oral teaching that includes gestures
     - Merely observing teachers' gestures can help students learn



Alibali & Nathan (2012)

Gesturing and Thinking

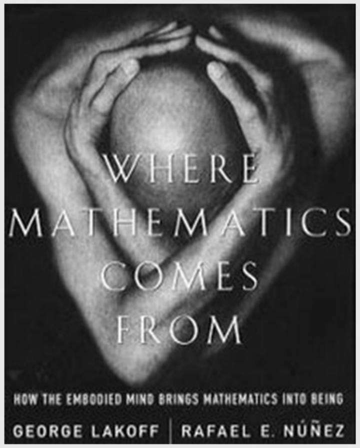
- **Implications teaching and learning:**
  - Promote gesture use vs. stopping finger use
  - Model gesture use



(Lakoff & Núñez, 2000)

Embodied cognition and mathematics

How the Embodied Mind Brings ***Mathematics*** *into Being*



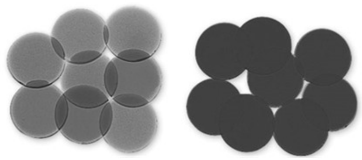
- **Embodied cognition**: Cognition is shaped by the motor system, the perceptual system, and the body's interactions with the environment
- **The embodiment of mind:** The detailed nature of our bodies, our brains, and our everyday functioning in the world structures human concepts and human reason.
- **Metaphorical thought:** For the most part, human beings conceptualize abstract concepts in concrete terms, using ideas and modes of reasoning grounded in the sensory-motor system.

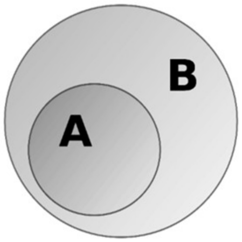
- **Mathematical *idea analysis***: analysis of mathematical ideas in terms of the human experiences, metaphors, generalizations, and other cognitive mechanisms giving rise to them.
  - Considers what structures of the mind allow it to do mathematics

Embodied cognition and mathematics:

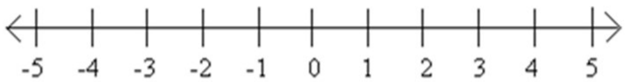
Conceptual Metaphors

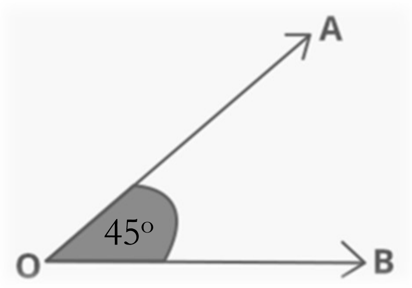
- **Grounding metaphors** yield *basic, directly grounded ideas.*
  - E.g. arithmetic as object collection/construction, measurement, sets as containers…





- **Linking metaphors** yield *sophisticated ideas* or *abstract ideas.*
  - E.g. numbers are points on a line, geometry (angles are numbers)…





Embodied cognition and mathematics:

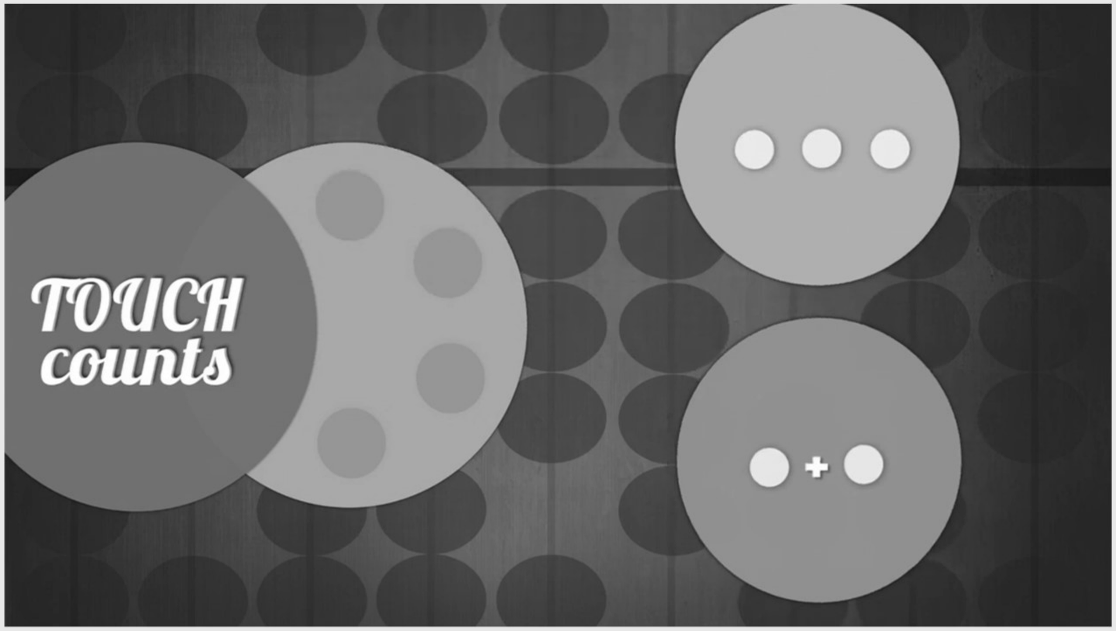
Numbers as collection or set

- What kinds of conceptual metaphors and bodily experience would support learning…?

(+2)+(-3) (+2)-(+3) (+2)-(-3) (-2)-(-3)

+ -

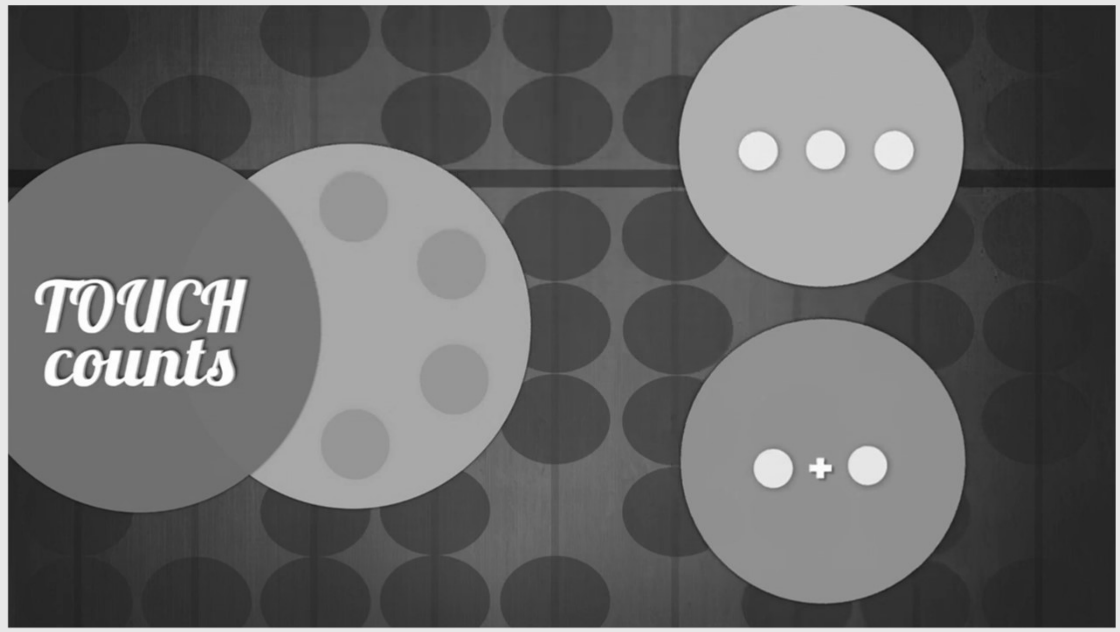
Embodied learning with TouchCounts:
Tapping, touching, seeing, hearing, making numbers



Embodied learning with TouchCounts:
Tapping, touching, seeing, hearing, making numbers

Transforming what mathematics is learned:

- One-to-one correspondence
- Ordinality ("put 10 on the shelf")
  - odd, even, and skip-counted number sequences (see: http://touchcounts.ca/numbers.html)
- Cardinality ("make 10"; "how many ways?")
  - "Which is biggest? Which is smallest?"
  - Adding *as*…
  - Partitioning *as*…
  - Equalpartioning *as*…
  - Multiplying *as*…
    - "make the largest product with your fingers"
    - commutative property: a*b = b*a
- Number as symbols



Transforming how it is learned:

- Tactile, visual & auditory feedback, improving finger perception

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(1) Gesturing and Thinking
Feedback – discussion – break

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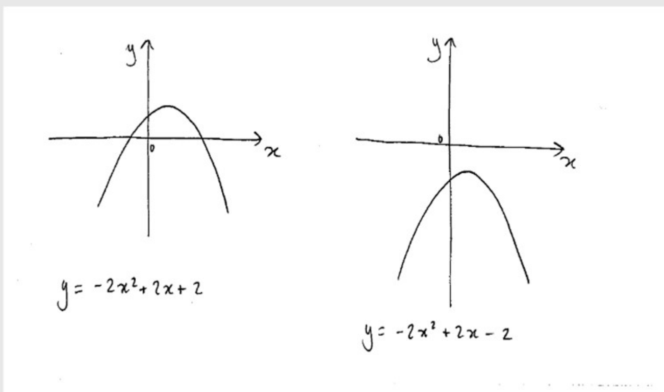
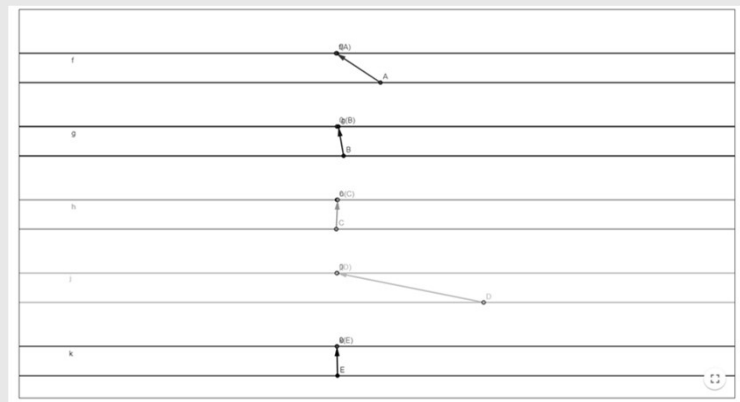
(2) Moving and Thinking

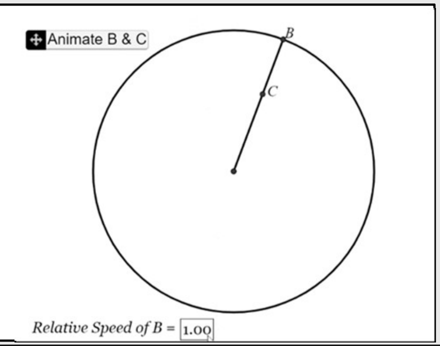
Motion and “e-motion”

https://www.geogebra.org/m/uMDXDnvu





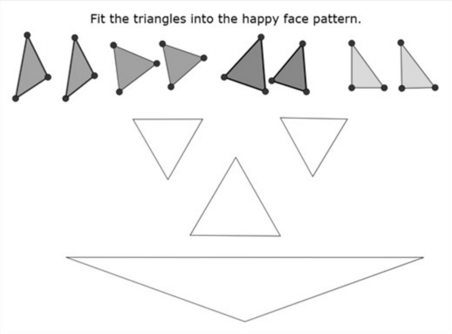
- **Descarte**: “I think, therefore I am”
  - Cartesian coordinates and graphs
  - Absence of motion and time
- **Sinclair**: “I move, therefore I am”(?)
  - Dynagraphs and “moving mathematics”
  - Embodied and engaging mathematics learning

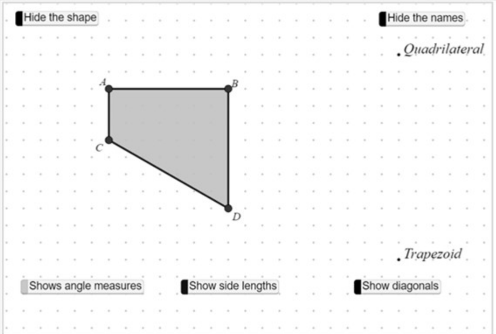


https://www.sfu.ca/content/dam/sfu/geometry4yl/sketchpadfiles/Petal/index.html

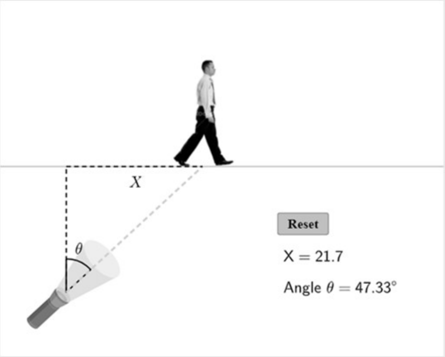
Embodied learning with Dynamic Geometry Environments (DGEs)

- **Interactivity: moving, dragging & feedback**
  - Example: Triangle and quadrilateral inclusivity

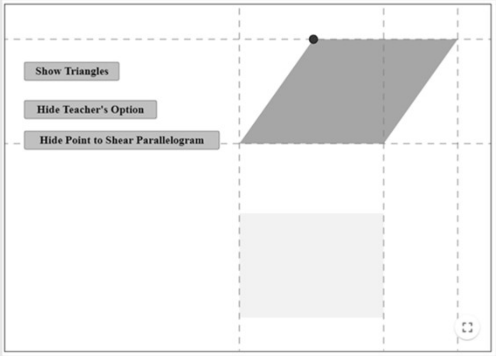




- **Temporality and continuity**
  - Example: Related rates

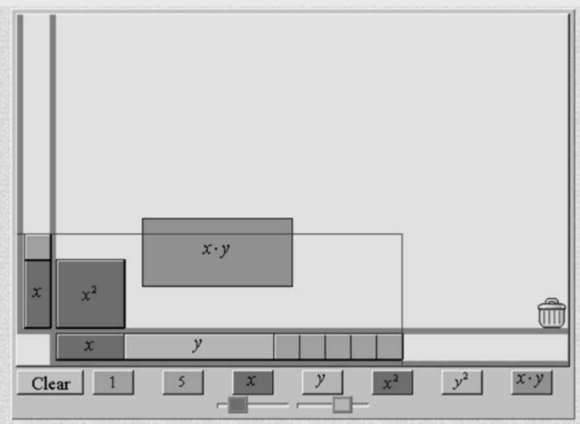


- **Visual and kinesthetic modes of learning**
  - Example: Shearing

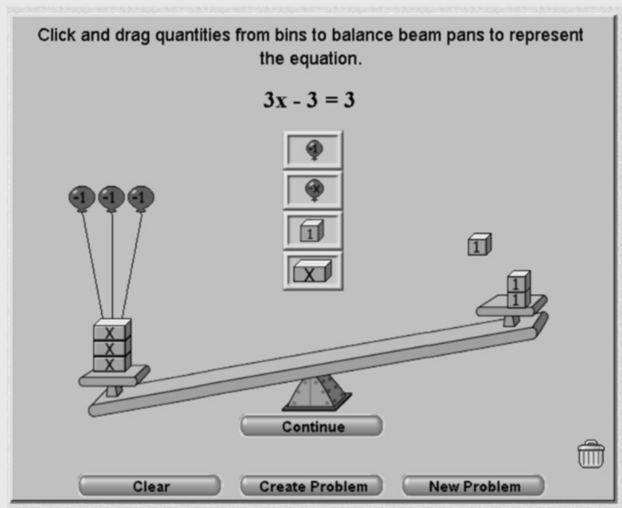


Action-based learning with virtual manipulatives (VM)

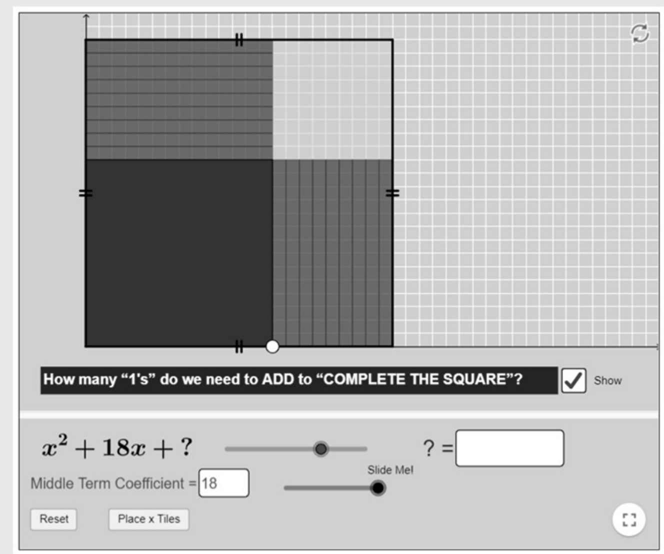
Algebra tiles



Solving equations



Completing the squares



http://nlvm.usu.edu/

https://www.geogebra.org/m/h25QAATA

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(2) Moving and Thinking

Feedback – discussion – break

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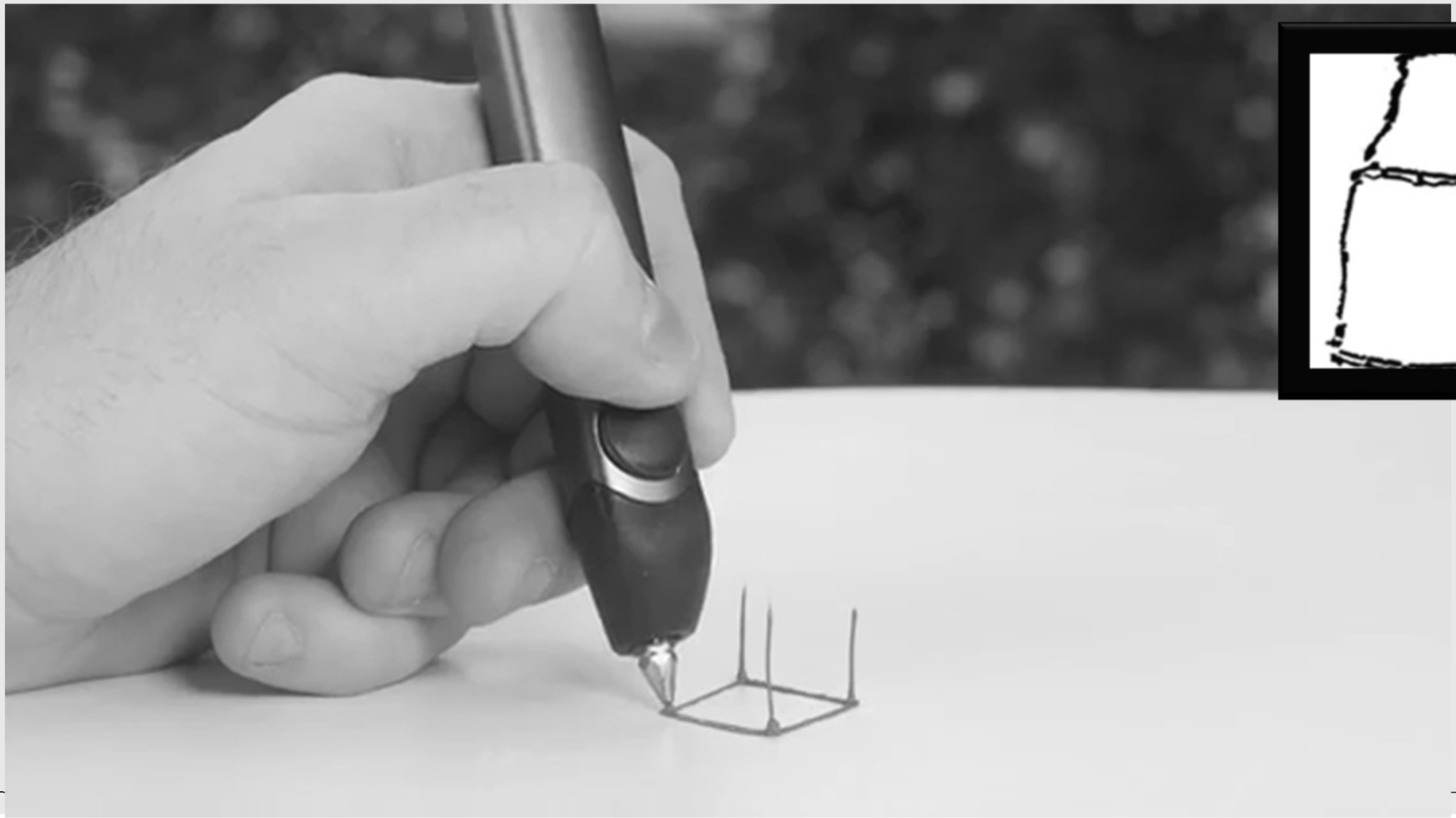
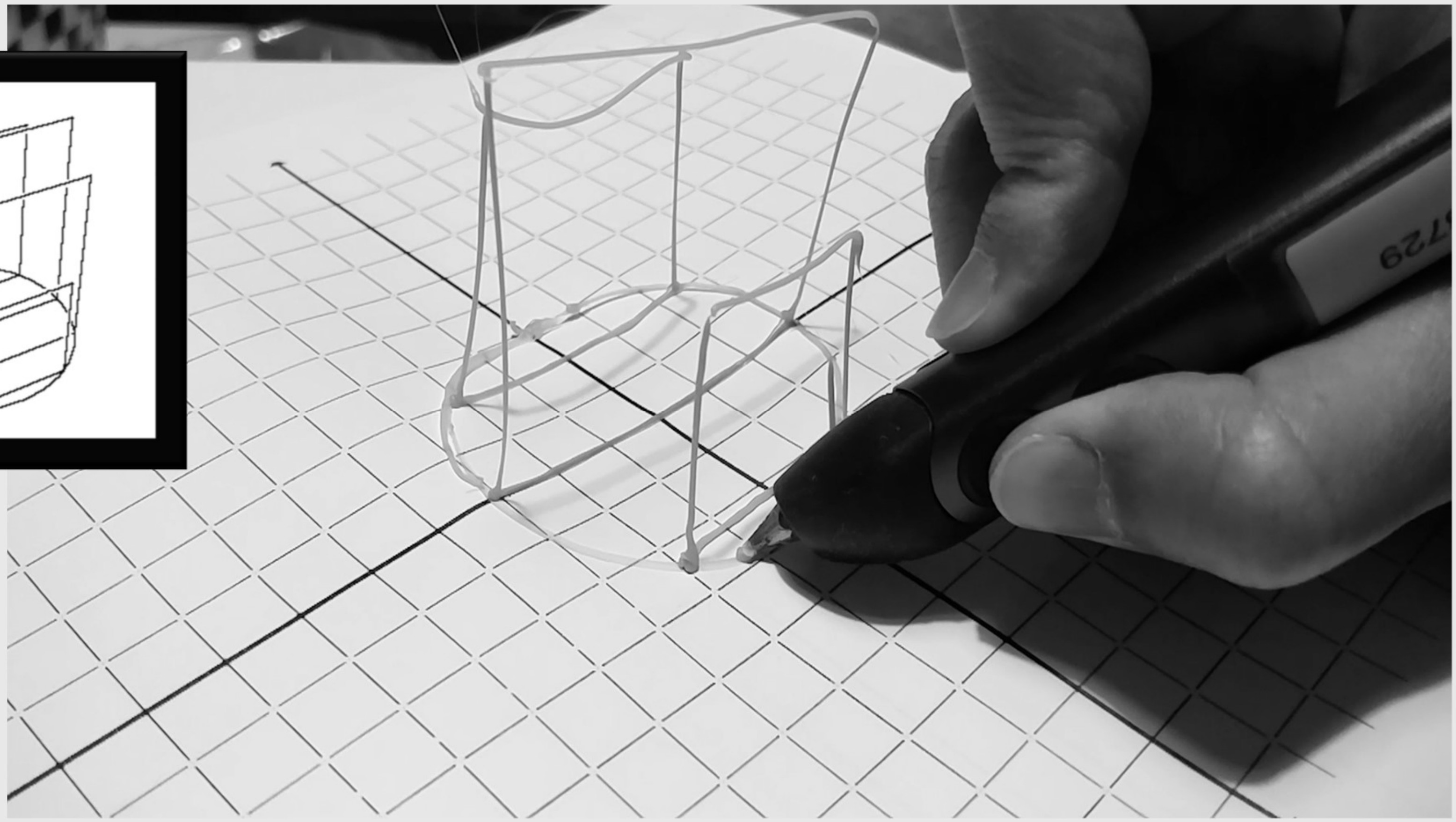
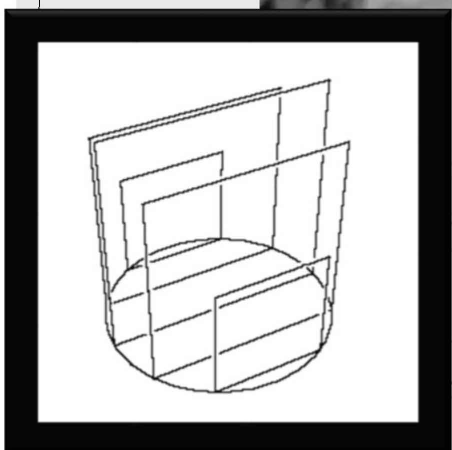
(3) Making and Thinking

3D Pen Demo 1: Expressing in 3D

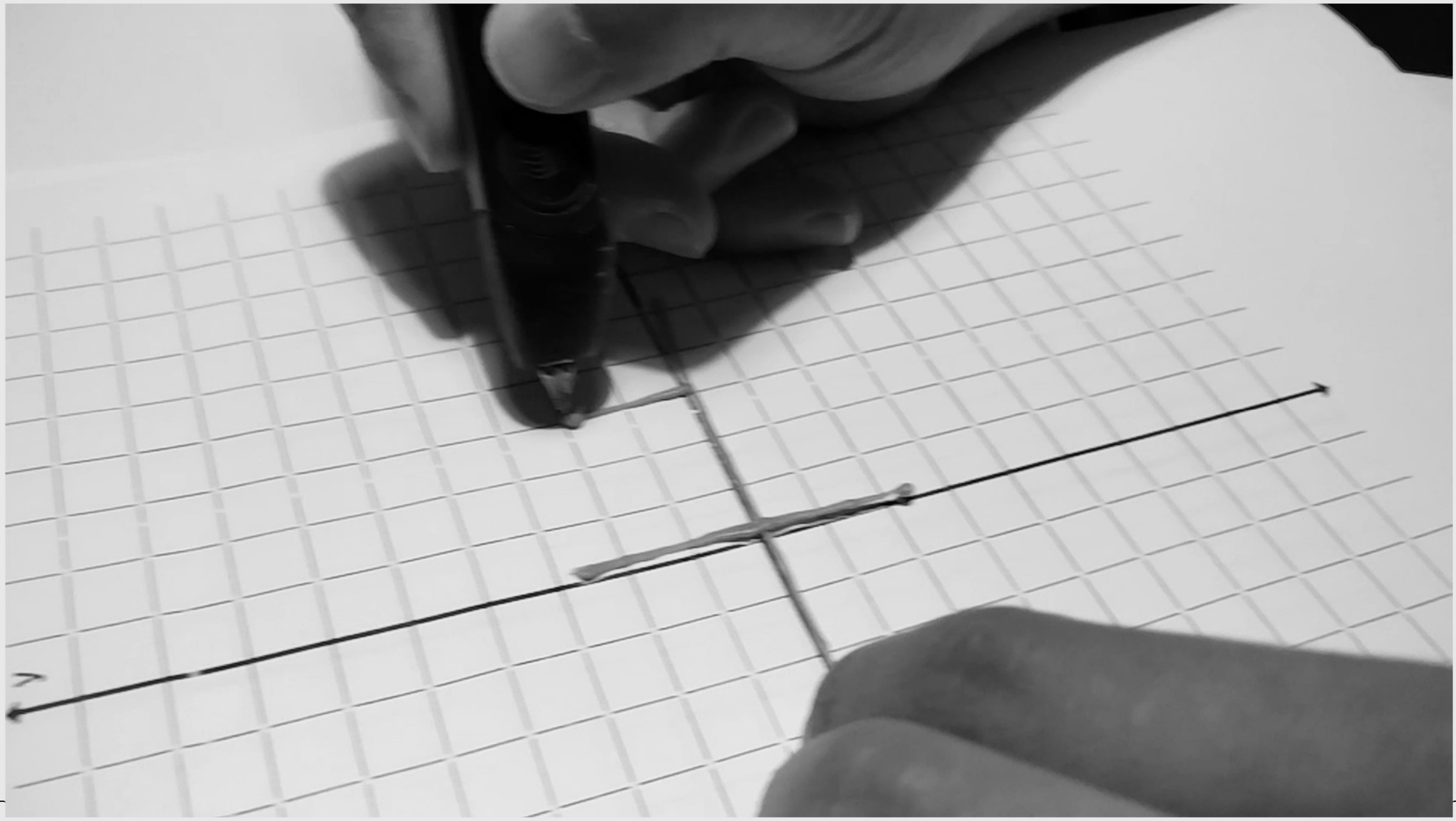


3D Pen Demo 2: Supporting visualisation





3D Pen Demo 3: New modes of thinking



Demo 4: The dual nature of diagrams & manipulatives

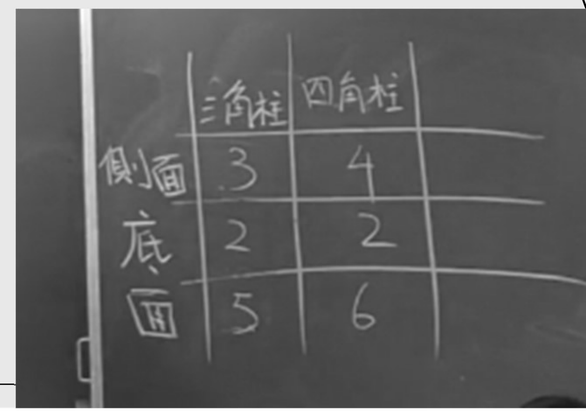
Primary School Geometry with 3D Pens

Ng, O., & Chan, T.H. (2017). Visualizing 3D solids with 3D printing technology. *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education, Vol 2 (p.30)*. PME: Singapore.

- Primary 5 and 6: Properties (faces, vertices, and edges) of different forms of prisms and pyramids. Students spent most of class time using 3D Printing Pens to draw different prisms and pyramids during the lesson. Adopting an inquiry and student-centered approach for investigating the target properties, the classroom teacher led a whole-class discussion on those properties.

Collaborative learning

Inquiry-based learning and generalization



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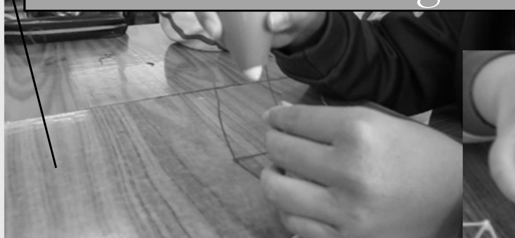
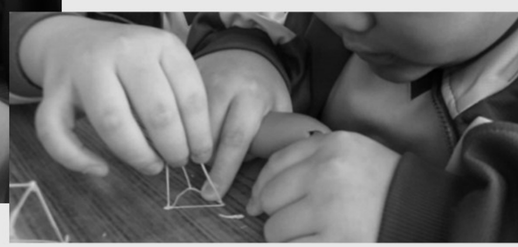
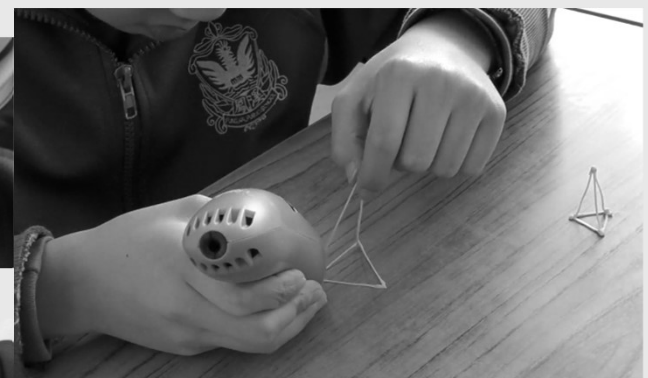
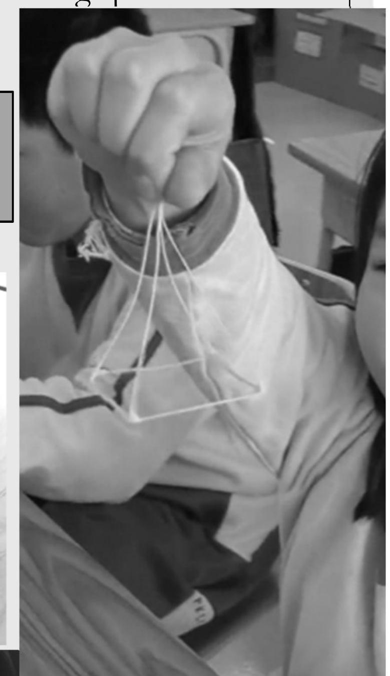
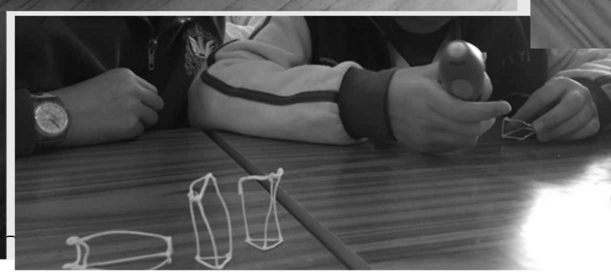
Deep learning vs. surface learning

Active learning and hands-on Making

Personal & meaningful

Cross sections of 3D solids

Towards a Materialist Vision of 'Learning as Making': the Case of 3D Printing Pens in School Mathematics

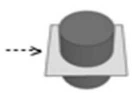
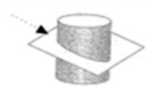
Oi-Lam Ng & Francesca Ferrara

International Journal of Science and Mathematics Education 18, 925–944(2020) | Cite this article

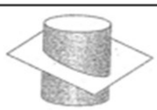
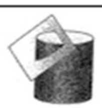
進行活動前先重溫一些重點

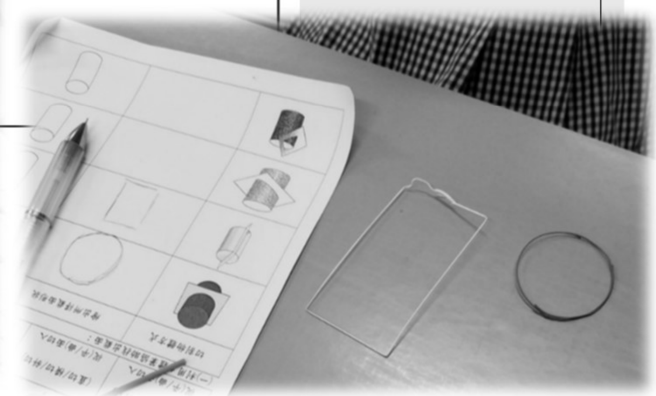
若果想在一個圓柱體製作截面，總的來說，可以從四個方向切入 ：

請圈出適當的切入方式及其切入面的特性：

			
(直切/橫切/斜切) 從(平/曲)面切入	(直切/橫切/斜切) 從(平/曲)面切入	(直切/橫切/斜切) 從(平/曲)面切入	(直切/橫切/斜切) 從(平/曲)面切入

活動(1)：

切割物體方式	繪出所得截面形狀	繪出與左圖相同截面的切割方式
		
		
		
		



Cross sections of 3D solids

Towards a Materialist Vision of ‘Learning as Making’: the Case of 3D Printing Pens in School Mathematics

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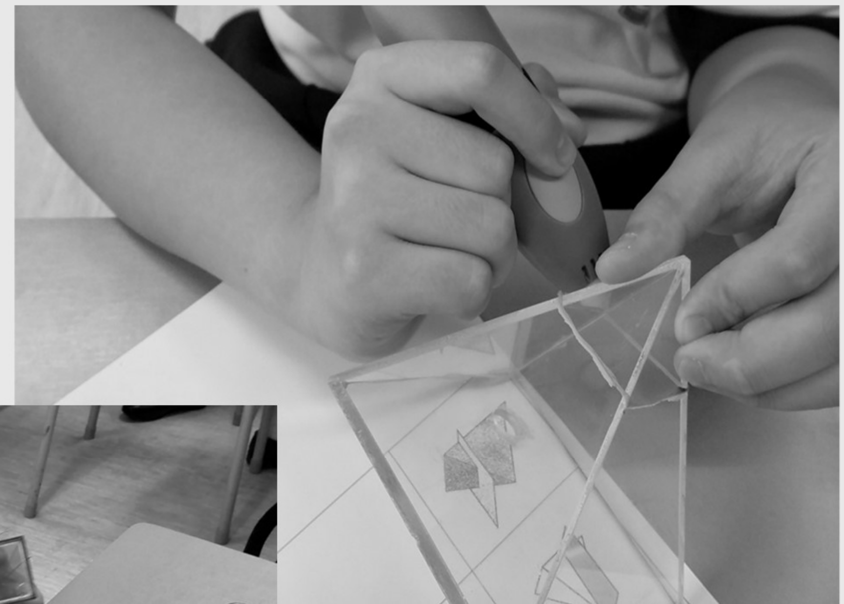
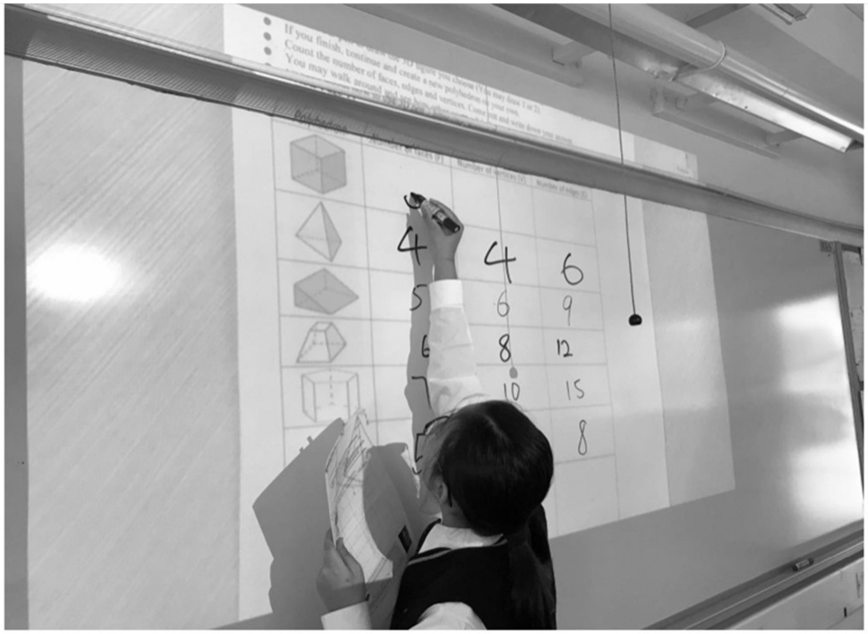
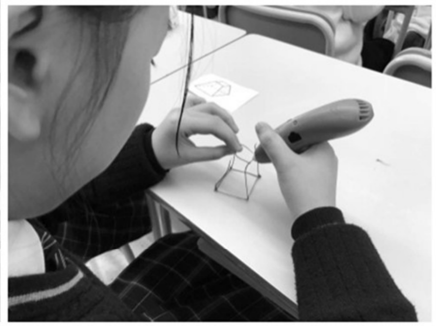
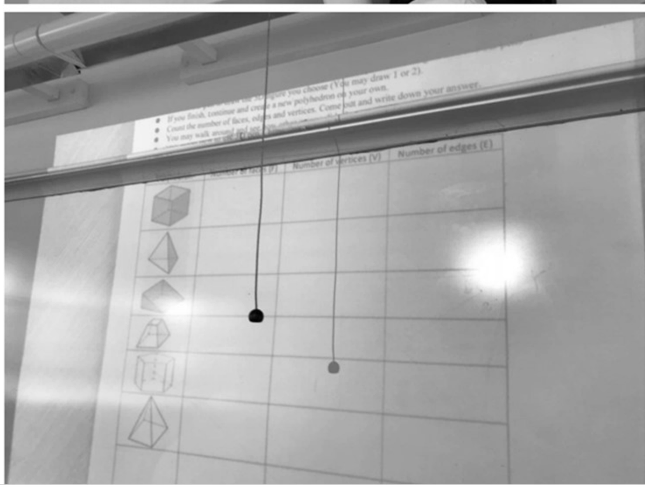
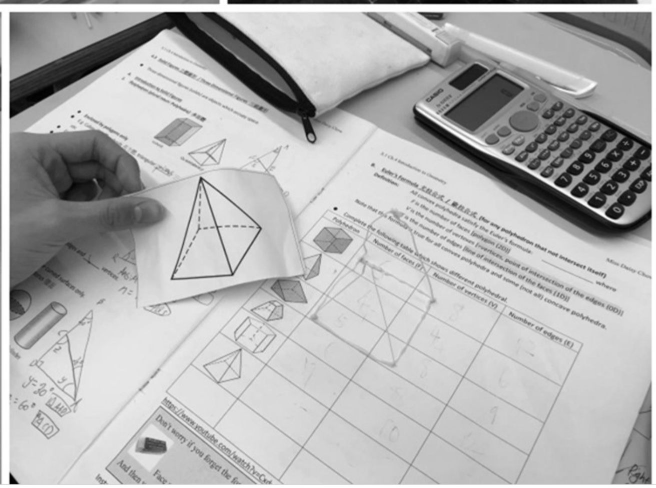








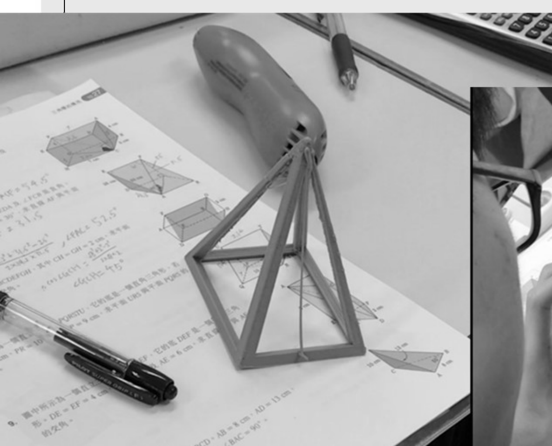


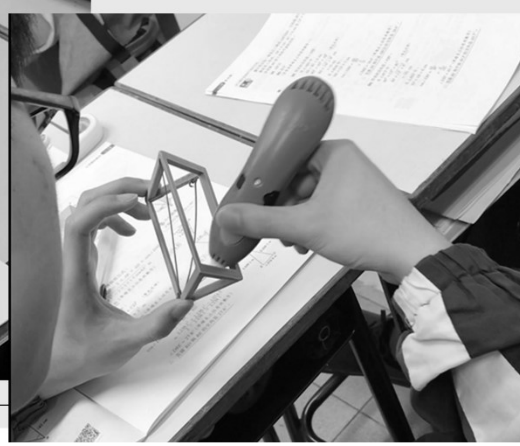
Euler Formula with 3D pens

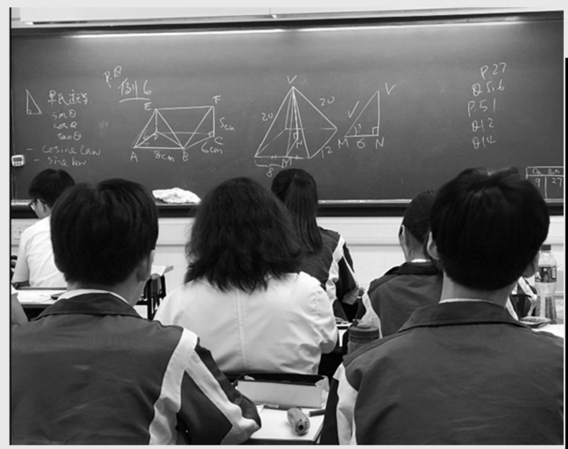
Active learning, hands-on Making, inductive reasoning, and making generalisation about Euler Formula (V+F-E=2) using the 3D Pens

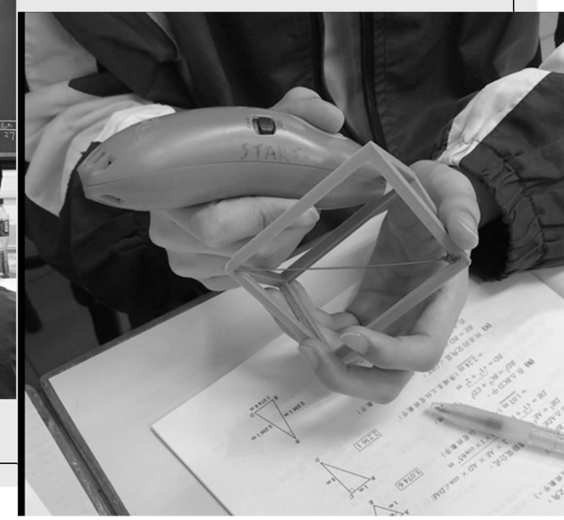
3D Trigonometry with 3D Pens

A series of lessons were designed to help Secondary 3-4 students visualize the angles formed (i) between lines, (ii) between a line and a plane, and (iii) between planes, which were found to be difficult to learn among secondary mathematics learners. These lessons incorporated both 3D printed models by a 3D printer, as well as students' hand-drawn "lines" using a 3D Printing Pens









Functions and Calculus with 3D Pens

Drawing in Space: Doing Mathematics with 3D Pens

Oi-Lam Ng and Nathalie Sinclair

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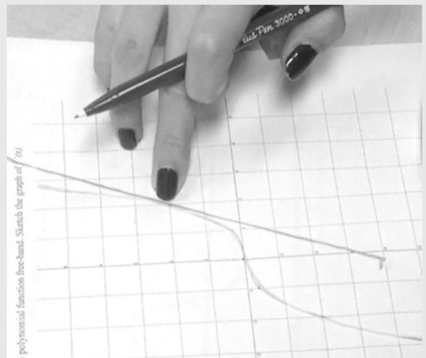
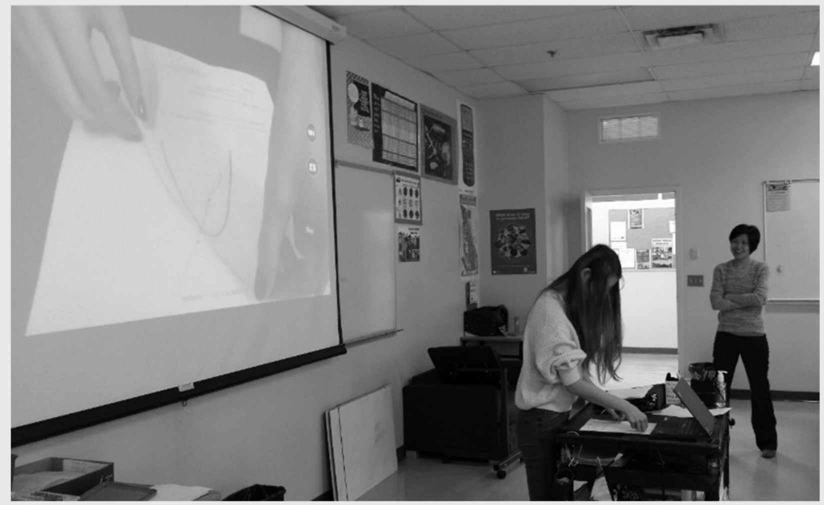
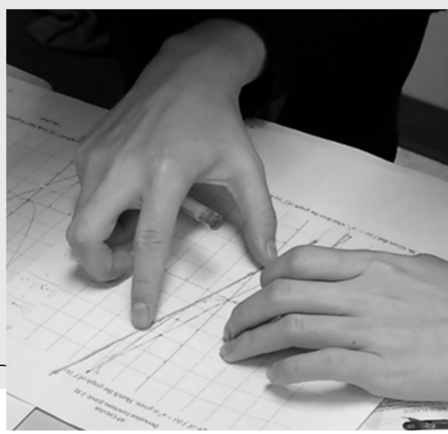
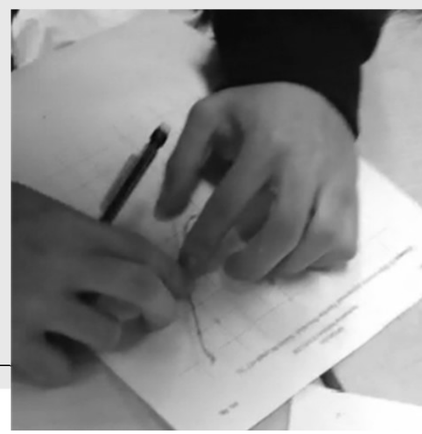
L. Ball et al. (eds.), *Uses of Technology in Primary and Secondary Mathematics Education*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-76575-4_16









New modes of exploring:

- graphs,
- tangents to a curve,
- derivative functions

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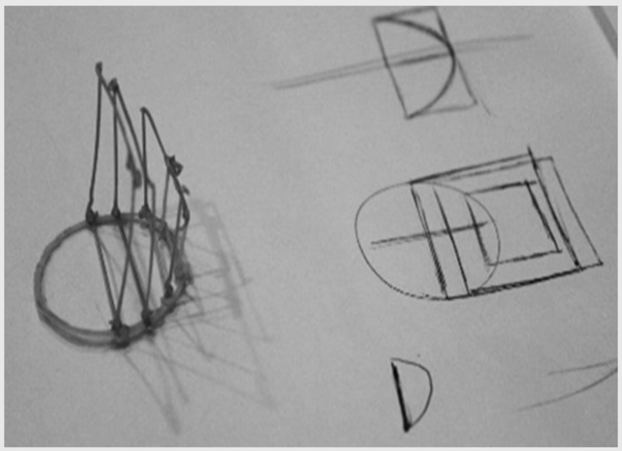
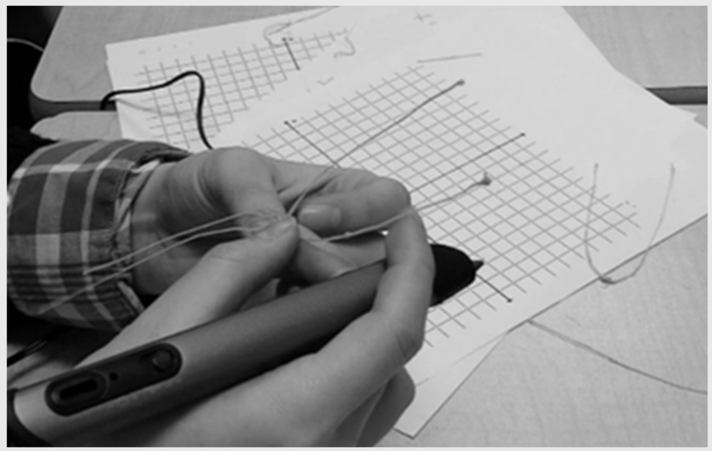
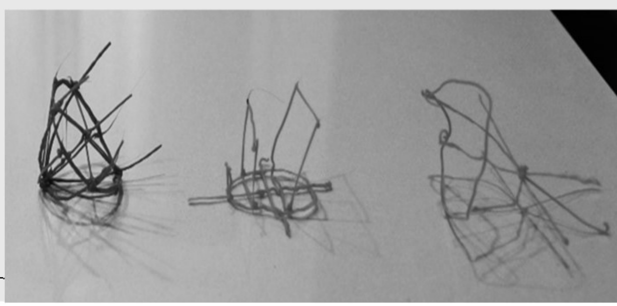






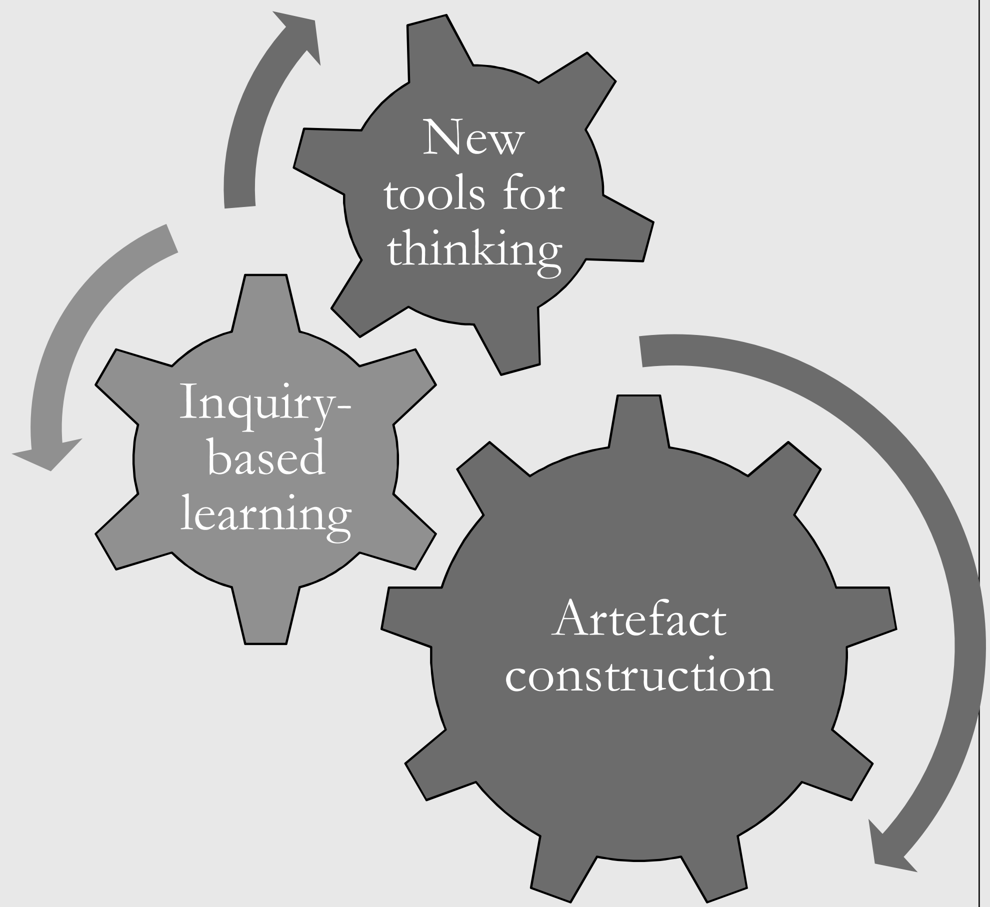
Supporting visualization of:

- Volume as definite integrals
- Solids of revolution

… in **active** and **embodied** ways

Conceptualization: Learning *as* Making

- Artefact construction
  - Active learning, hands-on production
  - Personal, purposeful, and flexible
  - Low entry, high ceiling tasks
- Inquiry-based learning
  - Experimentation and modelling
  - Inductive reasoning and generalization
  - Engineering design cycle
- New tools for thinking
  - Embodied and material interactions with concepts
  - Artefacts as tools



Recap and conclusion

- 將數學教育從線性、抽象和紙筆為主的特徵轉變為包含多種模式、動手實踐和應用現代技術的特徵。
- Technology can potentially transforms what and how mathematics is learned.

Abrahamson & Sánchez-García (2016):

"the pedagogical quality […] of action-based learning environments largely depends on […] what it means to learn a mathematical concept and what an instructor's role might be in this process. Thus, we are echoing Papert's consistent call to leverage the technological revolution as an opportunity for deep discussion of the potentially radical changes educational systems should undergo (Papert, 1993, 1996). Similar to Papert, we are optimistic that technological advances in educational media bear the potential of fostering students' deep understanding of mathematical concepts (p. 204-205)

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(3) Making and Thinking

Feedback – final discussion